# Statistical Analysis of Energy Distributions of Quiet Sun EUV Brightenings from High-Resolution Solar Orbiter Observations

*Caro Hermans (1,2), Cis Verbeeck (1), Nancy Narang (1), Daye Lim (1,2), Sergei Shestov (1,3), Frédéric Auchère (4), Marilena Mierla (1,5), David Berghmans (1), Antoine Dolliou (6), Wim Schoutens (2), Johan Segers (2), Emil Kraaikamp (1), Konstantina Loumou (1), Ewan Dickson (1), Bogdan Nicula (1), Luciano Rodriguez (1), Andrei Zhukov (1)*

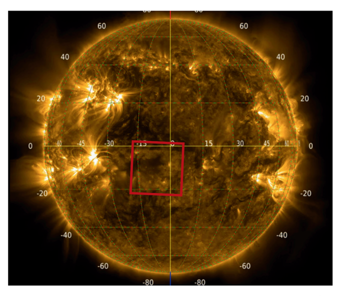
## Abstract

To address the long-standing **coronal heating problem**, we investigate the smallest observable transient events in the Quiet Sun using unprecedented high-resolution data from **Solar Orbiter's HRI EUV**. By applying a wavelet-based detection algorithm and incorporating **Differential Emission Measure (DEM)** analysis on SDO/AIA channels, we estimate the **thermal energies** of these small-scale EUV Brightenings.

Our statistical analysis employs **Maximum Likelihood Estimation (MLE)** to compare **power law** and **lognormal** models, utilizing refined methods from Clauset et al. (2009) and Corral et al. (2019) to constrain the lower energy bound. This robust characterization will provide new insights into the energy distribution of these events and their potential role in maintaining the solar corona's temperature.

## Data selection

### 2024-03-20T17:23:00 - 2024-03-20T18:22:55 (Quiet Sun)



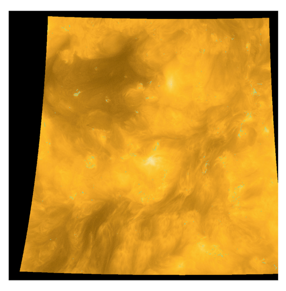
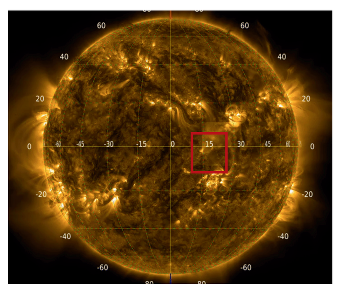


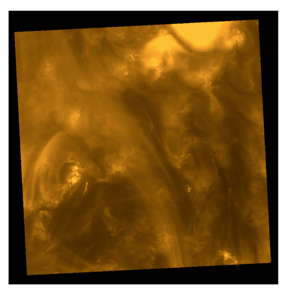
*Image in Carrington coordinates, cyan dots are detected EUV Brightenings*

- 5s cadence
- 1.87° SO-Earth Angle
- 0.4244 AU
- 3600 s

*Focus on thermal energy*

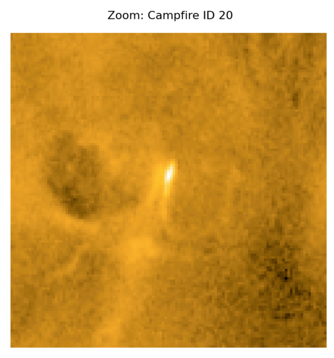
### 2023-04-04T06:18:08 - 2023-04-04T06:47:59 (Quiet Sun)





- 3s cadence
- 24.58° SO-Earth Angle
- 0.3255 AU
- 1794 s

*Focus on observed intensity*



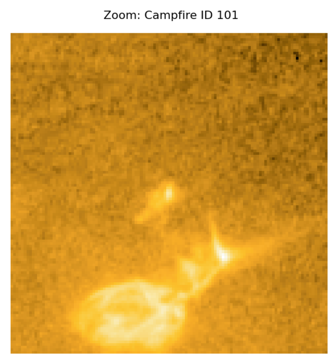




*Image field is 120 x120 pixels, which corresponds with 18.17 Mm x 18.17 Mm*

## DEM Calculations

**What is DEM?**
→ *Differential Emission Measure*

- y is an m-tuple corresponding to measurements by the AIA EUV channels

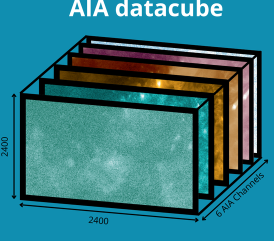
$$y_i = \int_0^\infty K_i(T) DEM(T) dT$$

$$y_i = \sum_{j=1}^{n} K_{ij} EM_j$$

- This can be written in the form $y = Dx$ for which we need to solve the corresponding L1 norm minimization problem:

*Minimize* $\|x\|_1$ *subject to* $y = Dx$

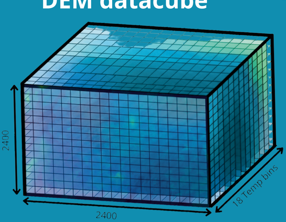
**AIA datacube**



***Basis Pursuit:***
- *long running time*
- *Divergent Points*

***DeepEM:***
- *short running time*
- *All points converge*

**DEM datacube**

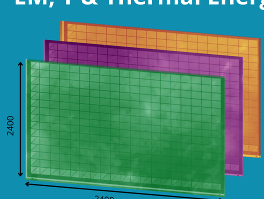


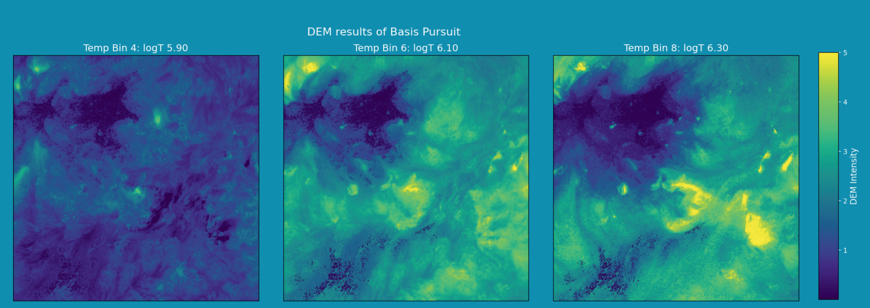
$$EM(i_C, j_C, k) = \sum_{n=1,\ldots,18} DEM(i_C, j_C, k, n)$$

$$T(i_C, j_C, k) = \frac{\sum_{n=1,\ldots,18} DEM(i_C, j_C, k, n) T_n}{\sum_{n=1,\ldots,18} DEM(i_C, j_C, k, n)}$$

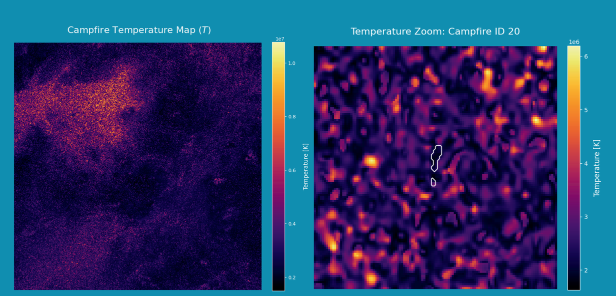
$$E_{th} = 3 k_B n_e T V$$

**EM, T & Thermal Energy**





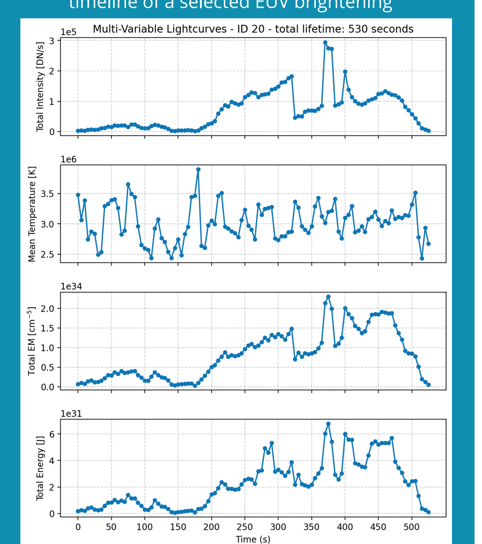

*From results of Basis Pursuit emcube: EM surfaces at 2024-03-20T17:23:00:228*




*From results of DeepEM emcubes: Temperature surface at 2024-03-20T17:23:00:228*

**Lightcurves over time**

Total intensity, temperature, EM and energy timeline of a selected EUV brightening




↓ *Summerize in Event Catalogue*

For each event ID: Mean, Peak, Max and total of variables over time

## Data preprocessing

1. Generate degradation-corrected Level 1.5 SDO/AIA data
2. Connect HRI EUV to AIA with account of light time delay
3. Align corresponding HRI EUV to AIA 171
4. Transform images to Carrington coordinates and crop fov of AIA to fov of HRI EUV
5. Detect EUV Brightenings
6. Associate EUV brightening masks with pixelwise thermal energy

## Scientific Context

Despite the Sun's surface temperature being only about 5,800K, the corona reaches temperatures exceeding $10^6$ K, suggesting a heating mechanism, this is called the **Coronal Heating Problem**

Flare energy distributions are typically modeled using a **power law**:

$$f(x) \sim x^{-\alpha}$$

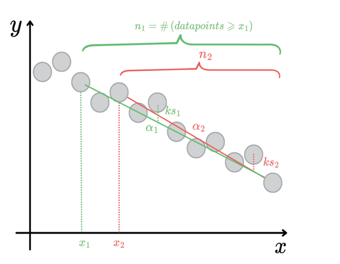
Flares contain enough energy to heat the corona if **$\alpha > 2$ ,** which is equal to the slope of the power law distribution seen on a log-log scale.

Today the **energy of flares remains insufficient** to heat the corona. We aim to estimate the contribution of EUV Brightenings in the Quiet Sun to coronal heating.

Power law slopes can only be estimated reliably by employing robust statistical methods. We use Maximum Likelihood Estimation and two schemes for estimating the lower bound for the model energy distribution. In view of evidence for lognormal behavior of flare energy distributions (Verbeeck et al. 2019), we investigate the merits of both power law and lognormal fits to the data. **Comparing the energy distribution of EUV brightenings to that of flares can provide strong hints whether they can be understood as one family or not.**

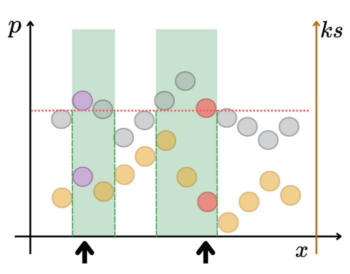
## Statistical Method

### Choose x_min, neglect all data below lower bound



**Clauset et al. 2009**

Based on minimum distance between empirical and theoretical model



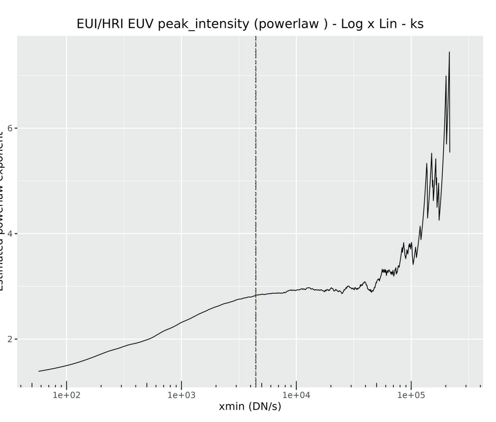
**Corral et al. 2019**

Based on generating synthetic datasets. For each option of lower bound, parameters of the distribution are fitted which leads to a generation of a distance distribution of the distance between the synthetic datasets. In its turn this leads to the calculation of a p-value from which we select the lower bound.

Standard choice and optimized for distance choice for threshold

### Check stability for choices of x_min

**Hill plots**




A Hill plot visualizes the Hill estimator against an increasing number of tail observations to identify the optimal threshold where the extreme value index stabilizes. For each choice of lower bound a parameter (e.g. exponent) can be plotted to assess the stability. The figure above is the Hillplot of the exponent of the intensity of the 2024-03-20 dataset.

### Compare different options for distribution

**Numerical comparison:**

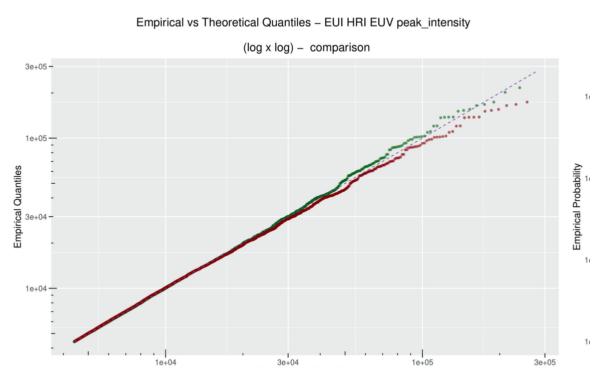
- Likelihood ratio
  - *comparing the goodness of fit of competing statistical models*

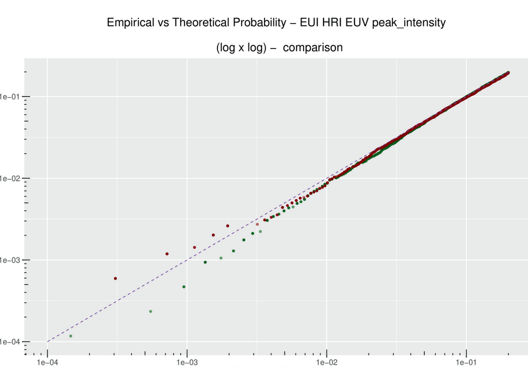
$$R = \frac{L_1}{L_2} = \prod_{i=1}^{n} \frac{p_1(x)}{p_2(x)}$$ *with p(x), probability density function*

- AIC & BIC
  - *Estimation of prediction error*
  - *Gives relative quality of statistical model*

**Visual comparison:**

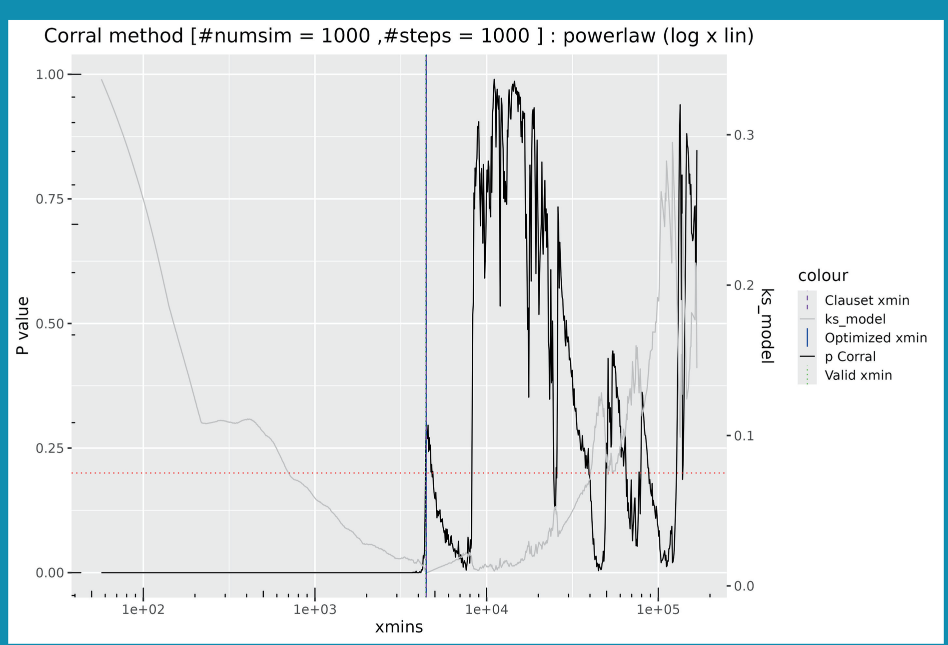
- QQ-Plot: Empirical v.s. Theoretical Probability
  - *Good if behaviour follows y = x*
  - *Compare power law and lognormal distribution*




- Also used: Histograms, CDF & CCDF

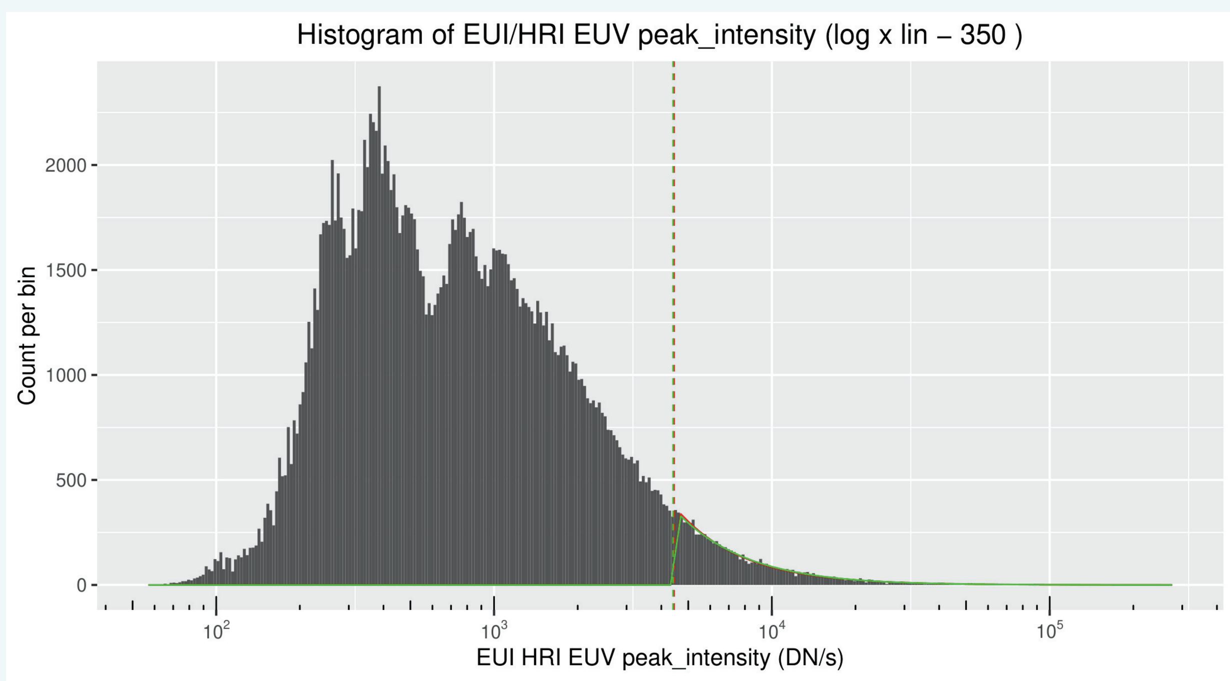
## Zoomed: Method of Corral et al. 2019

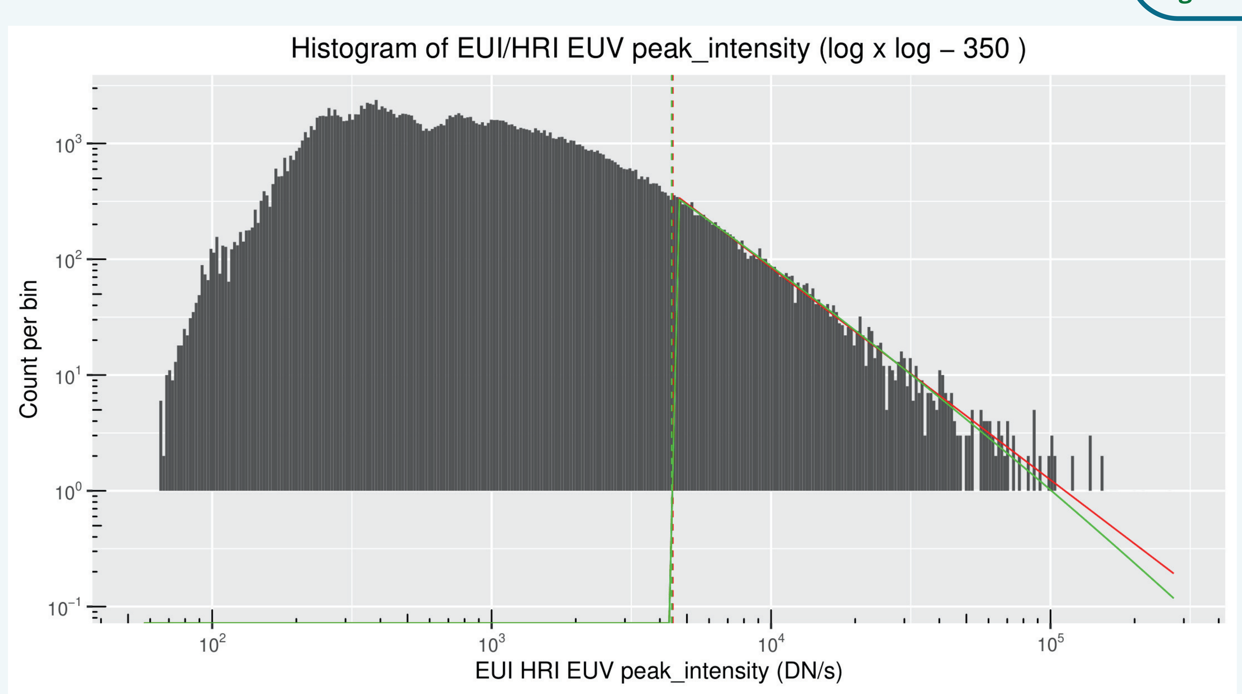



*In this plot we show the method of Corral et al. 2019 on the dataset of 2024-03-20. The vertical lines represent the different options to select the lower bound.*

## Work in progress: preliminary results

Power law
Lognormal




The histograms above represent the variable Peak Intensity of the 2024-03-20 data set. The plot on the left shows the log-lin scales. The vertical lines represent the choice of lower bound, in this case chosen by the method of Clauset et al. 2009. On the right, the plot with log-log scale is shown. This plot allows us to look at the tail in more detail.

## Affiliations

1: Royal Observatory of Belgium, Solar-Terrestrial Centre of Excellence
2: KU Leuven, Belgium
3: Centre Spatiale de Liège, Belgium
4: Institut d'Astrophysique Spatiale, France
5: Institute of Geodynamics of the Romanian Academy, Romania
6: Max Plank Institute for Solar System Research, Germany

## References

Berghmans, D., Auchère, F., Long, D. M., et al. 2021, A&A, 656, L4
Cheung, M. C. M., Boerner, P., Schrijver, C. J., et al. 2015, ApJ, 807, 143
Clauset, A., Shalizi, C. R., & Newman, M. E. J. 2009, SIAM Rev., 51, 661
Corral, Á. & González, Á. 2019, Earth Space Sci., 6, 673
Dolliou, A., Parenti, S., Auchère, F., et al. 2023, A&A, 671, A64
Hudson, H. S. 1991, Sol. Phys., 133, 357
Hortin, T. 2003, Ph.D. thesis, Univ. Paris XI
Joulin, V., Buchlin, E., Solomon, J., & Guennou, C. 2016, A&A, 591, A148
Verbeeck, C., Kraaikamp, E., Ryan, D. F., et al. 2019, ApJ, 884, 50
Wright, P. J., Cheung, M. C. M., Thomas, R., et al. 2019, Zenodo, doi: 10.5281/zenodo.2587015

## Contact

caro.hermans@kuleuven.be
caro.hermans@oma.be