Comparison of IAU2006/IAU2000a precession nutation model with VLBI observations.

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- The uncertainties and accuracies of lunisolar nutation components were quantified after a global analysis of 13 nutation series provided by different analysis centers.
- Fitted nutation amplitudes and a nutation series computed from the basic Earth parameters fitted on the C04 data series was obtained. The nutation residuals computed from the fitted nutation terms show a lower wrms than the values w.r.t. IAU2000A.
- From statistical approaches, we demonstrate that the tiny amplitude variations of the major nutation components are detectable from VLBI and can be interpreted in terms of physics of the Earth interior or external geophysical fluid effects.

The uncertainties of the lunisolar nutation components are quantified after analysis of 13 sets of independent global Earth's Orientation Parameters (EOP). The deviation of the long term nutation components to IAU2000A are consistent within the uncertainties suggested by previous studies but exceeding the errors determined in this work. The inconsistencies in the mid-term nutation components are partly explained by the indirect loading effects. The overall result indicates that the major nutation components are not constants as once thought, the millimeter level fluctuation in those terms are now detectable from VLBI determined global EOP solutions.

1 Introduction

Due to the gravitational attractions from celestial bodies, the Earth's rotation axis has various periodical motions w.r.t. its figure axis. These motions are named as nutations or Celestial Pole Offsets (CPO) in which the mean obliquity correction is applied. The forced nutations are expressed as a sum of harmonic components and most of them can be precisely modelled and predicted using the orbital information of the celestial bodies. The nutations as a part of Earth Orientation Parameters (EOP), are measured by various space based geodetic techniques namely VLBI, GPS, SLR, LLR etc. and the ground based instruments such as recently focused on Large Laser Ring facilities. The VLBI received radio signals from remote quasars are treated simultaneously at several data centers across the world, they are more or less following a similar procedure but using different software packages to retrieve the EOP. By combining a set of VLBI observations using Kalman filter or Allan variance analysis, the global EOP solutions are found. In addition, the International Earth Rotation and Reference Systems Service (IERS) and International VLBI Service (IVS) provides the service to combine the individual EOP solutions and publish the global EOP on a regular base to the scientific communities.

Although the torque induced by the Moon, the Sun and the other planets are precisely known, the response of the global Earth to these external torques (Moon, Sun and Planets) are slightly deviated from a pure solid body due to the complicate geophysical process such as: the seasonal mass circulation of atmosphere and ocean, the deformable mantle, the presence of a liquid outer core and a solid inner core etc. In practice, the initial precession and nutation model is constructed based on a solid Earth model. The IAU1980 nutation series are released from an elastic rotational ocean-less Earth model. The precision of all those series are truncated at the ⍰ μ*as* level. With the cumulated VLBI observations, the discrepancy between the observation and the rigid Earth nutation series IAU1980 is significant. It can reach two orders of magnitude higher than the uncertainties of the VLBI measurement itself. The deviation to the IAU1980 nutation series is investigated from the VLBI measured Earth's nutation, using the covariance analysis method and the least square analysis. The corrections of the major 21 nutation terms has been computed w.r.t. IAU1980 nutation model and the secular trends has been estimated in longitude and in obliquity w.r.t. IAU1977 precession rate [*Herring et al.*, 2002] from three VLBI solved global EOP solutions (GSF, USNO, IAA) and a combined series from these three which are covering the observation period from 1980 to 2000. The standard deviation of the nutation amplitudes with a period shorter than 400 days is 5μ*as* and 38μ*as* for the period longer than 400 days when it is compared with the MHB2000 nutation series inferred from a set of Basic Earth Parameters (BEPs) [*Mathews et al.*, 2002]. Then the amplitudes of the nutation components are obtained by convolving a designed transfer function with a rigid Earth model REN2000 [*Souchay et al.*, 1999]. This series is adopted by IAU as IAU2000 nutation model. The correction to the procession rate was found w.r.t. MHB2000 values by [*Capitaine et al.*, 2003], it is updated as the IAU2006 procession model. The IAU2006/IAU2000A procession-nutation model is composed with 678 lunisolar and 687 planetary terms induced by the gravitational attraction from the Moon, the Sun and the other planets, respectively. As all of the components are a function of time, each term can be computed or predicted at any given time stamp.

However, if we assume the IAU2000A nutation series is accurate enough to account for the cumulated VLBI observations, then it is possible to numerically quantify the uncertainties of each nutation component by fitting the extended observation to this series.
In this work, the amplitude of lunisolar nutation component (ALNC) is fitted to the latest EOP provided by various independent analysis centers. By an explicit comparison among the 11 independent EOP series together with their combination, plus the IVS (IVS15q2e) and the IERS (14EOPC04) combined series, an effort to quantify the uncertainties of the current nutation series was made and the possible geophysical interpretations the source of errors were discussed.

2 Fitting IAU2000A nutation series from VLBI determined EOP using the iterative method

The figure axis in space has two type of motion: the procession and nutation. The Cartesian coordinates of the pole of the Earth's axis in the geocentric celestial reference frame are:

$$X(t) = [\psi_A(t) + \Delta\psi(t)]\sin \epsilon \ (1)$$

For a short period, approximation is made by letting procession, $\psi_A = const.$ and replacing sin by mean obliquity at J2000 sin $_0$ = 0.39777716, Then the elliptical motion can be decomposed into circular motions [*Dehant and Mathews*, 2015]:

$$\Delta\psi_n(t) = \sin{}_0(\Delta\psi_n^{ip}\sin\Xi_n(t) + \Delta\psi_n^{oop}\cos\Xi_n)(t) \ (2) \text{ and } \Delta\epsilon_n(t) = \Delta\epsilon^{ip}{}_n\cos\Xi_n(t) + \Delta\epsilon^{oop}{}_n\sin\Xi_n(t) \ (3)$$

After applied the mean obliquity correction (sin $_0$ = 0.39777716) to the changes in longitude, it becomes the *dx* of celestial pole offset (CPO), the changes in obliquity is *d*y of CPO. The observed CPO can be described by a harmonic function:

$$dx - C_x = A_s\sin(\Xi_n(t)) + A_c\cos(\Xi_n(t)) \quad (4)$$

$$dy - Cy = B_c\cos(\Xi_n(t)) + B_s\sin(\Xi_n(t)) \quad (5)$$

where, *dx* and *d*y is observed CPO, for simplicity the motion of each component was kept as the original value from IAU2000A $C_x = dA_s/dt \sin(\Xi_n(t))$, $C_y = dB_c/dt \cos(\Xi_n(t))$, suppose a rectangular matrix: $A = [\sin(\Xi_n(t)) \cos(\Xi_n(t))]$; Then equation 4 and 5 can be transformed to a linear system, $Ax = b$; where $x = [A_s A_c]'$,

$$dx - C_x = Ax \quad (6)$$

let $Ay = c$; where $y = [B_c B_s]'$,

$$dy - C_y = Ay \quad (7)$$

The *b* and *c* are the sum of VLBI measurements and the C_x and C_y, *A* is the coefficient matrix, can be calculated from the argument number. The unknowns are the *x* and y. The size of matrix *A* is $m \times n$, where *m* is the total number of observed CPO and increasing with time, *n* is the number of nutation components, here *n* is equal to 678 × 2 for the lunisolar components. The amplitude matrices (*x*,y) can be solved through iterative algorithm. The iterative method is usually applied to solve square asymmetric systems; under-determined consistent systems; over-determined systems and regularized systems. As $m \leq n$ thus this is a typical over-determined systems. The rank of matrix is computed using the singular value decomposition (svd) meth. A synthetic time series was generated from 20,000 to 80,000 days in Modified Julian Date with one MJD step. For the contribution of lunisolar origin, it forms a coefficient matrix *A* with a dimension (120002 × 1356). The rank (*r*) of *A* is equal to 1324. The rank of coefficients matrices (*A*) is equal to the augmented matrices ($A|b$) but smaller than the total number of unknowns (1356), therefore, the system is consistent but *x* and y has infinite solutions. In order to find the most likely solutions of the system ($A|b$), an iterative algorithm, the Lanczos method was applied. Lanczos method is used to solve the eigenvalue and eigenvector approximation through iterative algorithm process. The optimized solution is found after a series of trail, supposing:

$$(A - \mu B)x = b \quad (8)$$

After the $(A - \mu B) = LDL^0$ is factorized by finding triangular L and diagonal D, and then one solution, *x* could be solved by,

$$x = L^{-1\,'} D^{-1} L^{-1} b \quad (9)$$

Let, $B = CC^0$ according to the Lanczos method, C may be a rectangular but is assumed to have linearly independent columns. equation 8 became,

$$(A - \mu I)^{-1} = C'(A - \mu B)^{-1}C \quad (10)$$

The solution of *x* and y are searched through a range of values [μ_0, μ_1, ..., μ_n], at each trial, a residual series is computed with a set of intermediate values, the final results were found until the weighted-root-mean-scattering (w*rms*) is converged and the numerical solutions of *x* and y corresponding to the minimum w*rms* values are selected as the final results.

3 VLBI determined global Earth Orientation Parameters

The data used in this work is coming from two sources: the IERS-14EOP-C04 through the IERS and the rest of global EOP solutions were obtained from online archives of the Crustal Dynamics Data Information System (CDDIS), NASA. In order to obtain the EOP solutions from the original time delay measured in a given VLBI baseline and a certain session observation, each analysis center has to project the location of the antenna to the ITRF and ICRF, then correcting the local environmental effects.

In summary, the selection of VLBI observation sessions, the software and the way to correct the local environmental effects by each analysis center, are different. However, in term of nutation model, all the analysis centers is albeit to IAU2000A mode. This suggest that the discrepancy in the ALNC derived from different centers could be originated from two sources: the inconsistence in the data treatment and the bias of the nutation model.

4 Results

The correction terms from VLBI fitted nutation series with respect to the *IAU1980* nutation series were found from each EOP solutions and compared with the *IAU2000A*'s values. The uncertainties on the results were computed on a period from 1990 to 2017 and their scattering corresponds to the standard deviation (SD) of the differences between each of 14 series and *IAU*2000*A* values. Due to higher uncertainties in the Celestial Pole Offset (CPO) determination prior to 1990, the amplitudes of each nutation component were fitted on the observations after 1990. The SD of the long period (> 400 days) nutations terms are less than 20 µ*as* estimated from 11 independent CPO residuals, plus the combined CPO and the IVS15q2e and 14EOPC04 products, its deviation is about 30 µ*as* to 40 µ*as* to IAU2000A nutation model. The standard deviation of the mid period term (100 < *p* < 400 days) is < 25 µ*as* and the short period term (< 100 days) is < 10 µ*as* which is suggested by the analysis of all the data file. The 18.6 yrs nutation terms periodic motion may influence the secular trends estimated from the CPO residuals and vice versa because they are related to the same parameters, so very much correlated.

5 Discussion and Conclusions (you could add some prospective discussions in this section)

The basic Earth's parameters have been inferred from the VLBI determined EOP in frequency domain [*Mathews et al.*, 2002] and in time domain [*Koot et al.*, 2008], a comparison of BEPs derived from both approaches were compared using IERS-08EOPC04 solutions. The differences in the real and imaginary parts of coupling constants at CMB may be explained by different coupling mechanisms and by the consideration of a non-hydrostatic flattening of the core [*Dehant et al.*, 2017]. We have, in this paper, performed this determination again with the idea of trying to evaluate the accuracy of the results.

We have analyzed 11 sets of individual global Celestial Pole Offsets (CPO) solutions plus their combinations, the IVS15q2e and the IERS 14EOPC04. The differences are related to the fact that each CPO solution is obtained using different software and following various ways to correct local environmental effects from the original observations. As usually in data analysis fitting parameters using an a priori model, there are uncertainties (related to the data analysis) and accuracies (related to the real differences between the true model and the observation). These accuracies are only available when the model represents the reality. Uncertainties are usually underestimated, which is not wanted when performing a geophysical interpretation of the data. It is thus very interesting to try to estimate the distance between the accuracies and the uncertainties. This can be performed by using a measure of these differences based on a statistical approach, using the $\chi^2(f)/f$, where f is the number of samples considered in the computation and $\chi^2(f)$ represents the goodness of a fit determining if a sample of f data matches the real population. In other words, $\chi^2(f)$ tells you either if your sample of f data represents the data you would expect to find in the actual population or if two series of f data samples are related or differ from each another. For f =1, one considers the latest sample and the computation represents the present; for f = the total number of data, one considers the cumulated effects going back in the past. As for nutations, the present-day observation are more accurate than the first ones, the χ^2/f should be more performant for f small. For further independent evaluation of the accuracy, we than performed a Monte Carlo approach (MC) considering random sampling in the series, independent of the timing. Ratios with respect to the ideal case of $\chi^2(f)/f$ at f=1 should thus be lower than 1. We have shown that the MC for the $\chi^2(f)/f$ are ranging from 1*e*5 to 1*e*3 and that the ratios with respect to the ideal case are falling in a range of [0.8, 1.0] for the 13 CPO residual series. There is one series as minimum value, the IERS series, and one as maximum values, the IVS series, the 11 other CPO residual series falling in between. The differences between the two are at the 1.3% level with a w*rms* = 24.5 µ*as*. The values for 11 other EOP have a 1.15% with the w*rms* = 14.2 µ*as*. The EOP solved with the SOLVE/CALC SD have a 0.91% fluctuation in $\chi^2(f)/f$, corresponding to the w*rms* = 5 µ*as*. The EOP solved by the OCCAM SD have a 1.03% with the w*rms* =12.2 µ*as*. The differences among the EOP solutions solved by several independent analysis software are much lower than the total amount of uncertainties of the nutation components, which suggests that there is no significant inconsistence originated from various ways to retrieve the nutation residuals from VLBI measured time delay. However, the combined IVS shows a highest χ^2/f (f = 1) value indicating an overestimated uncertainties and the lowest χ^2/f (f = 1) obtained from IERS sets suggesting the uncertainties may underestimated.

The significant deviations between the IAU2000A nutation series and the VLBI fitted results are remaining significant in the long period nutation 18.6 y*rs* and 9.3 y*rs* components at the level of 40 µ*as*, 2 times over the associated errors, but consistent in the uncertainties estimated by [*Herring et al.*, 2002]. In addition, the annual and semiannual nutations show a same level of deviation than the long period ones comparing to the IAU2000A series. But if the loading contributions derived from the weather model are considered, the gap between VLBI fitted nutation series and the IAU2000A model values can be reduced, but still could not be closed. In addition, the free core nutation (FCN) mode has an unpredicted contribution to the signal and its frequency is close to the annual nutations; it is thus better to be studied the annual contribution to nutation together with the FCN contribution. It must be mentioned that the indirect atmospheric loading contributions are determined from the weather data over a period of 2004-2013, while the fits of VLBI data covering the period of 1984-2016. Furthermore, the 4-daily samples of atmosphere momentum were used previously, while there is now an increase to 6 points or more in serval analysis centers. Thus, the atmosphere loading effects to the nutations will more precisely be determined in a near future. However, it is interesting to quantify the contributions of indirect loading (Atmosphere, Ocean and Hydrology) at a regular base together with the amplitudes of the FCN as it is catalogued by *Lambert* [2006].

The uncertainties of sine and cosine coefficients of the lunisolar nutation are derived from the 11 independent EOP solutions and their combination plus two combined series IERS14EOPC04 and IVS15q2e using the iterative Lanzos method. Nutation residuals computed from combined series, IVS and IERS sets, w.r.t. our fitted series shows an overall 30 µ*as* improvement than the w*rms* calculated w.r.t. IAU2000A for the complete data set.

We have re-estimated the scattering effects from short window (40 measurements in each window) data as well as window of two years. The length of the window did not produce any significant differences between CPO residuals computed w.r.t. new fitted series and the IAU2000A series. But both results shows a better agreement between the formal uncertainties and CPO residuals comparing to the previous studies [*Herring et al.*, 2002]. The improved values of the scattering effects could be attributed to the increased number of VLBI sessions and the improved quality of the original measurements. In addition, our analysis proofs that the new series has a better agreement with the long and mid period terms and a same level of accuracy with the short terms.

The amplitudes and phases of free FCN contribution are changing with time since long time and serval studies suggests that these variations may be explained by the changes of atmospheric angular momentum [*Lambert,* 2006]. However, after globally fitting all the nutation terms, we found that the amplitudes of several major nutation terms may also vary with time because of the atmospheric loading effects or even fluid core varying contributions. Therefore, the repartition of individual source of indirect contributions determined from certain window of VLBI measured CPO may not be valid for the up-to-date complete data sets. Their time variables if considered as atmospheric effects alone, could shield topography effects at the core-mantle boundary, coupling mechanisms, and the energy dissipation at the depth of Earth’s core.

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References

[1] Capitaine N., Wallace P.T., Chapront J., 2003, Expressions for IAU 2000 precession quantities. Astronomy and Astrophysics, 412, 567-586, DOI: 10.1051/0004-6361:20031539.

[2] Lambert S.B., 2006, Atmospheric excitation of the Earth's free core nutation. Astronomy and Astrophysics, 457, 2, 717-720, DOI: 10.1051/0004-6361:20065813.

[3] Mathews P.M., Herring T.A., and Buffett B.A., 2002, Modelling of nutation and precession: new nutation series for non-rigid Earth and insights into the Earth’s interior. J. Geophys. Res., 107(B4), CI: 2068, DOI: 10.1029/2001JB000390.

[4] Souchay J., Loysel B., Kinoshita H., and Folgueira M., 1999, Corrections and new developments in rigid Earth nutation theory: III. Final tables `REN-2000´ including crossed-nutation and spin-orbit coupling effects. Astr. Astrophys. Supplement, 135, 111-131, DOI: 10.1051/aas:1999446.

[5] Wahr J.M., 1981, The forced nutations of an elliptical, rotating, elastic and oceanless Earth. Geophys. J. Roy. Astron. Soc., 64, 705-727.