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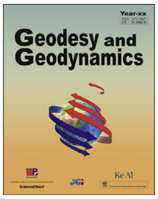

Basic Earth's Parameters as estimated from VLBI observations

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The global Very Long Baseline Interferometry observation for measuring the Earth rotation's parameters was launched around 1970s. Since then the precision of the measurements is continuously improving by taking into account various instrumental and environmental effects. The MHB2000 nutation model was introduced in 2002, which is constructed based on a revised nutation series derived from 20 years VLBI observations (1980–1999). In this work, we firstly estimated the amplitudes of all nutation terms from the IERS-EOP-C04 VLBI global solutions w.r.t. IAU1980, then we further inferred the BEPs (Basic Earth's Parameters) by fitting the major nutation terms. Meanwhile, the BEPs were obtained from the same nutation time series using a BI (Bayesian Inversion). The corrections to the precession rate and the estimated BEPs are in an agreement, independent of which methods have been applied.

1. Introduction

Due to the gravitational attractions from celestial bodies, the Earth's rotation axis has various periodical motions w.r.t its figure axis. These motions are named as nutation. The forced nutations can be expressed as a sum of harmonic components and precisely modeled and predicted using the orbital information of celestial bodies [1]. Earth's nutations can be directly measured using VLBI (Very Long Baseline Interferometry), which were initiated around 1970s [2]. VLBI is a technique in which radio telescopes hundreds to thousands of kilometers apart observe the same radio source (Quasars) in the sky. After the digitized signals are combined at a central dedicated data processor (the correlator), the time delays between two antennas, are extracted and corrected from local environmental effects. After combining the individual VLBI observation around the world using Kalman filter, a set of EOP (Earth's Orientation Parameters) are obtained, which including polar motions (dX, dY), length of day, $UT1-UTC$, changes in longitude and obliquity. However due to the fact that the real Earth's behavior deviates from an elliptical, oceanless, elastic Earth with a fluid outer core and solid inner core earth model [3]. Therefore, the nutation residuals are remaining large if one compares the observations with the predictions based on this model. On the other hand, the VLBI measured nutations have been analyzed in detail by Refs. [4,5]. After fitting those results to a set of equations explicitly allows for mantle anelasticity, inner core dynamics, and non-hydrostatic equilibrium effects [6], has obtained a set of BEPs (Basic Earth's Parameters) using the LSQ (least squares method). Based on these BEPs and REN2000 [1], a nutation model is built, latter on it is recommended by IAU as a nutation reference model IAU2000 [7]. The MHB2000 predicted nutation series for the 1980–2000 time interval with periods <400 days are in good agreement (5 μas) with VLBI observations but longer period nutations (>400 days) show deviations up to 56 ± 38 μas [5]. Since the cumulated VLBI observations have been doubled and the quality of the data are also improved comparing to previous studies. We present, in this work, results from a reprocessing of the VLBI IERS-EOP-c04 w.r.t. IAU1980 (hereafter will be refereed as EOPc04) over the 1984–2015 time interval using a least square method to fit the amplitude of the complete nutation series. The BEPs were then estimated from this series using the LSQ method in the frequency domain. Meanwhile, the BEPs were estimated again using the BI (Bayesian Inversion) with the same VLBI measured nutations in the time domain.

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2. Data and analysis

We analyzed EOPc04 (International Earth Rotation and Reference Systems Service, Earth Orientation Parameters) data sets consist of 11,658 sessions 24 h VLBI global solutions. It covers the period from January 01, 1984 to December 01, 2015. EOPc04 are smoothed values at 1-day intervals w.r.t. IAU1980 [8]. The data sets are open access through ftp://hpiers.obspm.fr/iers/eop/eopc04/. The amplitudes of nutation series could be derived from VLBI observations through two approaches: one is to use LSQ analysis to get the amplitudes of individual component [9], the other is through Fourier analysis to isolate different nutation terms [10,11]. In this work, we applied LSQ method to fit 32 years VLBI observed nutations to get the amplitudes of the nutation terms. Nutations can be observed in the changes of longitude:

$$\Delta\psi(t_j) = \sum_j \left[\Delta\psi_{sj}\sin(\omega_j(t)) + \Delta\psi_{cj}\cos(\omega_j(t))\right] + \Delta\psi(t) \qquad (1)$$

and in the changes of obliquity:

$$\Delta\varepsilon(t_j) = \sum_j [\Delta\varepsilon_{cj}\cos(\omega_j(t)) + \Delta\varepsilon_{sj}\sin(\omega_j(t))] + \Delta\varepsilon(t) \qquad (2)$$

where ω_j is the angular speed of the j^{th} periodical component, t is time. s stands for sin part, c is cos part. ω_j is determined based on astronomical observations [1]. $\Delta\psi(t)$ and $\Delta\varepsilon(t)$ are the procession rate corrections to the longitude and obliquity, respectively. The period of Earth's precession is about 26,000 years, thus its behavior is nearly linear even at the decadal time scale. For the nutation residuals w.r.t. IAU1980, the additional correction to the processing rate and some nutation terms are necessary (Fig. 1). Here, the corrections to the precession rate were fitted together with the nutation terms. The VLBI observed nutation series was reconstructed by adding the nutation residuals w.r.t. IAU1980 to the IAU1980 model values. In the IAU1980 convention, it contains 106 nutation terms. The amplitude of each term is truncated at 0.1 mas level [12]. Before starting to fit the observed EOPc04, the part induced by the planetary gravitational attractions was removed by computing theoretical values using the RDAN98 [13]. It could contribute several hundreds of μas in the nutation residuals if not properly modeled, which corresponds to a non-negligible displacement in longitude and obliquity as well (Fig. 2). After removing it, we fitted 11,658 sessions EOPc04 VLBI global solution and obtained coefficients of all nutation terms, but here we only listed the terms which are predicted and given in Ref. [6] (Table 1). Since the MHB2000 nutation theory is using VLBI fitted nutation series as an input, it is important to update the fitted series before estimating the BEPs. Therefore, the 'input' of MHB2000 model is also compared with. In this work, the VLBI fitted series used by MHB2000 model is named as HMB series [5]. To be noticed that the HMB VLBI fitted series were not exactly same as the nutation series inferred from the BEPs according to MHB2000 theory. In this paper, we are focused on the comparison of nutation components which were predicted from the BEPs in MHB2000 and those same components fitted using the VLBI observations by HMB.

In general, our fitted results are in a good agreement with both MHB2000 predicted nutation terms and HMB fitted ones (see Fig. 3). The 1σ value of our fitting is given and the χ^2 is computed from residuals w.r.t. MHB2000 and w.r.t.HMB, respectively. The freedom of χ^2 is one (Tables 1 and 2). For the 18.6 years nutations, the real part of fitted results are consistent within their uncertainties with respect to MHB2000 values or HMB fitted results, but our results are closer to MHB2000 predicted values for the 18.6 years terms. For the annual and short period nutations, our fitted results had better agreement with HMB VLBI fitted results than the MHB2000 predicted values (Table 1). Globally, the uncertainties are remaining larger for the imaginary part, no matter which results were compared with (Table 2). There is an exception, a weak nutation term with the period 1615.75 days, our fitted results and HMB results are consistent but significantly deviated from the values given by MHB2000 (Table 1). A comprehensive comparison with IAU2000A precession nutation mode is under going with various EOP solutions. Considering the amplitudes of this term is really small, it impacts on nutation series prediction is negligible.

However, in order to further exam the improvement of the new nutation series on the nutation modeling, we calculated the WRMS

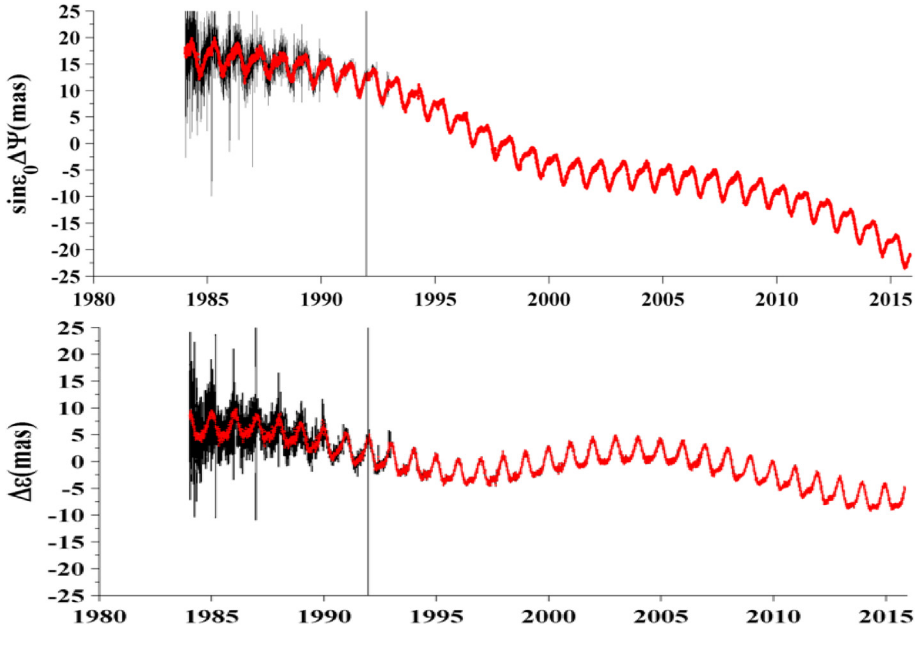

Fig. 1. IERS-08-EOP-C04 nutation residuals w.r.t. IAU1980 convention. The black solid line is the uncertainties in the VLBI global solution, red dots are 1-day smoothed nutation series. The vertical line marks the date January 1st, 1992. After this date, the uncertainties of the VLBI solutions is nearly complete buried in the signals.

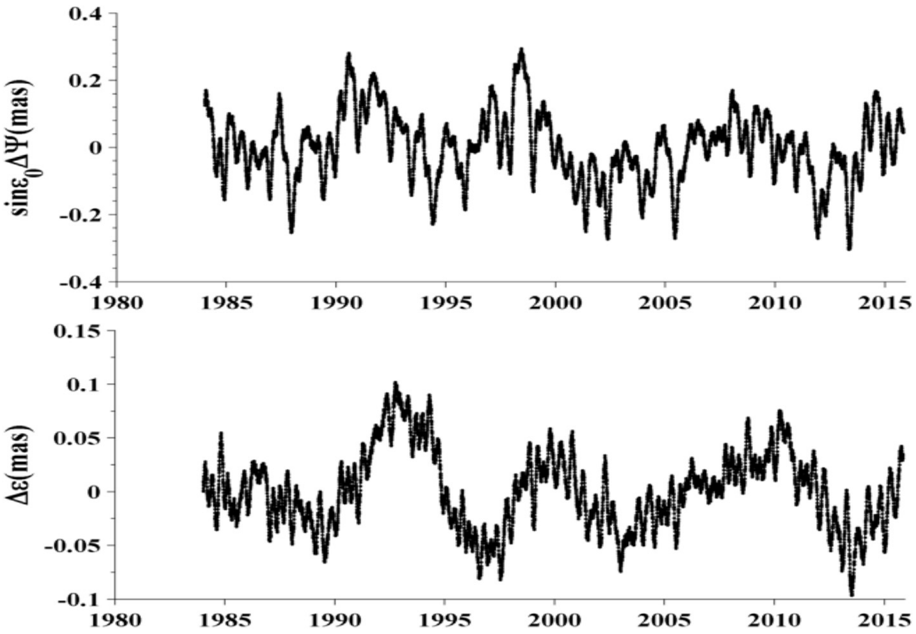

Fig. 2. Nutations from planetary contributions were computed with the RNDA98 catalogue.

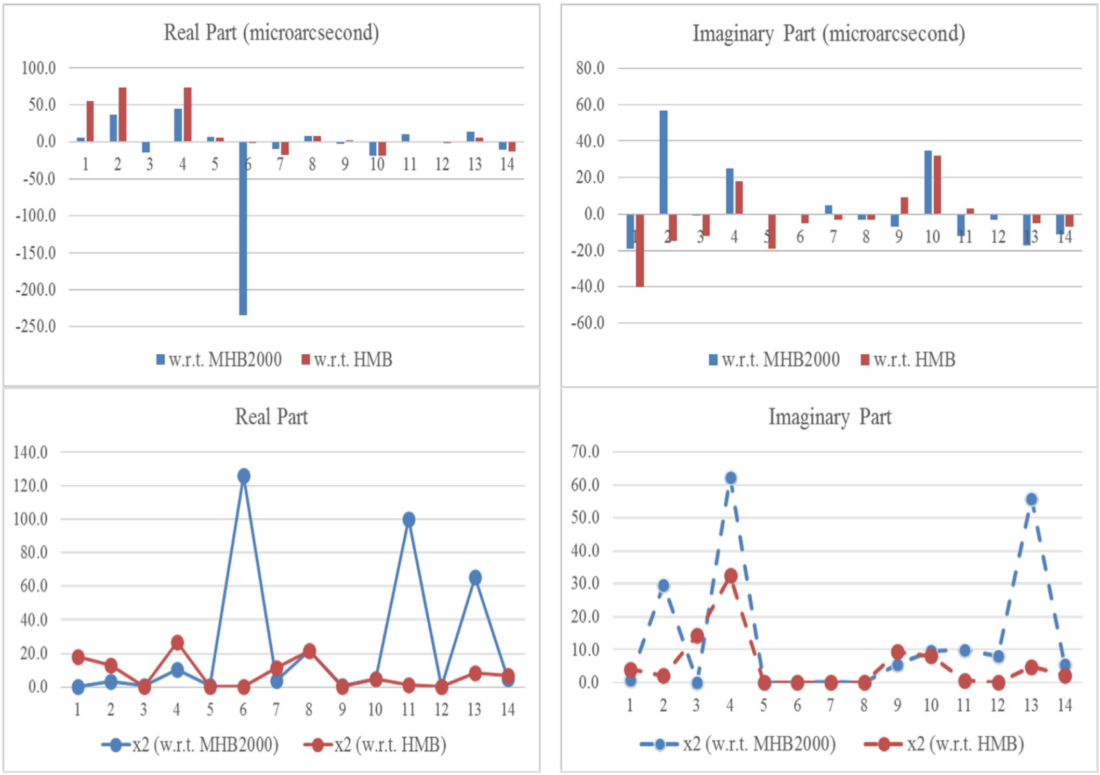

Fig. 3. The VLBI fitted nutation terms in this work and its comparison with the values of MHB2000 and HMB. The χ^2 are plotted together.

(weight-root-mean-square) scatter with three different subsets. The WRMS is defined as [9,4]:

$$\mathrm{WRMS} = \left[\sum_{j=1}^{n} \delta a_j^2 \Big/ \sigma_j^2 \Bigg/ \left(\sum_{j=1}^{n} 1 \Big/ \sigma_j^2 \right) \right]^{1/2} \tag{3}$$

where δa_j, is the residual for the nutation correction, σ_j is the uncertainties of the global VLBI solutions, n is the number of measurements. The results are given in Table 3. The magnitude of WRMS is depending on two factors, one of which being the uncertainties of the global VLBI solutions which is taken from the EOP files. Since it is a subject more related to the measurement uncertainties of each VLBI station and the performance of global solver software, we will not discuss it here but all related

Table 1
The real part of Nutation series derived from 32 years VLBI observations and it is compared with the values listed in MHB2000 and the same components in HMB VLBI fitted series.

No.	Period	$\times 10^3$(μas)	Res.$_{MHB}$(μas)	Res.$_{HMB}$(μas)	σ(μas)	χ^2_{MHB}	χ^2_{HMB}
1	−6798.38	−8024.770	5.0	55.0	13.0	0.1	18.0
2	+6798.38	−1180.423	36.0	73.0	20.4	3.1	12.9
3	−3399.19	86.121	−14.0	0.0	16.9	0.7	0.0
4	+3399.19	3.659	45.0	73.0	14.1	10.2	26.8
5	−1615.75	0.001	6.0	5.0	8.5	0.5	0.3
6	+1615.75	−0.107	−234.0	−2.0	20.8	126.4	0.0
7	−365.26	−33.057	−10.0	−18.0	5.3	3.5	11.3
8	+365.26	25.653	8.0	8.0	1.7	21.7	21.7
9	−182.62	−24.566	−3.0	2.0	3.7	0.7	0.3
10	+182.62	−548.490	−19.0	−19.0	8.5	5.0	5.0
11	−27.56	−13.797	10.0	1.0	1.0	100.0	1.0
12	+27.56	14.482	0.0	−2.0	4.3	0.0	0.2
13	−13.66	−3.634	14.0	5.0	1.7	65.3	8.3
14	+13.66	−94.209	−11.0	−13.0	4.9	4.9	6.9

Table 2
The imaginary part of Nutation series derived from 32 years VLBI observations and it is compared with the values listed in MHB2000 and the same components in HMB VLBI fitted series.

No.	Period	$\times 10^3$(μas)	Res.$_{\mathrm{MHB}}$(μas)	Res.$_{\mathrm{HMB}}$(μas)	σ(μas)	χ^2_{MHB}	χ^2_{HMB}
1	−6798.38	1.414	−19.0	−40.0	20.0	0.9	4.0
2	+6798.38	−0.048	57.0	−15.0	10.4	29.8	2.1
3	−3399.19	−0.029	−1.0	−12.0	3.2	0.1	14.4
4	+3399.19	0.026	25.0	18.0	3.2	62.5	32.4
5	−1615.75	0.000	0.0	−19.0	0.0	0.0	0.0
6	+1615.75	0.000	0.0	−5.0	0.0	0.0	0.0
7	−365.26	0.336	5.0	−3.0	15.5	0.1	0.0
8	+365.26	0.128	−3.0	−3.0	10.7	0.1	0.1
9	−182.62	−0.050	−7.0	9.0	2.9	5.7	9.3
10	+182.62	−0.467	35.0	32.0	11.3	9.6	8.1
11	−27.56	−0.047	−12.0	3.0	3.8	10.0	0.6
12	+27.56	−0.002	−3.0	0.0	1.1	8.1	0.0
13	−13.66	−0.030	−17.0	−5.0	2.3	55.9	4.8
14	+13.66	0.113	−11.0	−7.0	4.6	5.6	2.3

Table 3
Weighted root mean square scatter was calculated for different nutation residual subsets w.r.t. our nutation series and MHB2000 (values in the round brackets).

Subsets	1984–2000 [5844 sessions]	1992–2015 [8735 sessions]	1984–2015 [11658 sessions]
$\sin\varepsilon_0\Delta\psi$(μas)	189.8 (206.7)	189.2 (209.4)	190.9 (201.0)
$\Delta\varepsilon$(μas)	185.3 (194.0)	186.9 (196.4)	190.8 (207.8)

information could be found from various publications [2]. Here we took it directly from the EOP files. The other factor which is influencing the WRMS is the accuracy of the nutation model which has been used to compute nutation residuals. It is interesting to notice that the nutation residuals computed w.r.t. our VLBI fitted nutation series, are 10 μas better than the WRMS calculated w.r.t. MHB2000. This is true for all three subsets.

3. BEPs (Basic Earth's parameters) estimated from the partly updated nutation series

We estimated the nutation series by LSQ analysis of VLBI observed nutation residuals EOPc04. From the fitted nutation series, we further inferred the BEPs following MHB2000 theory [6]. At the same time, the BEPs have been estimated directly from the VLBI residuals EOPc04 using the Bayesian Inversion. The details about the BI method, we refer to the work of Ref. [14]. The advantage of the BI is that it is independent of the prior model and it can easily take into account all the information in time domain [14]. In addition, we applied the LSQ analysis of BEPs following the approach of [6]. The VLBI measured nutations can be described by a linearized dynamic equation:

$$M(\sigma)x(\sigma) = \phi(\sigma)y(\sigma) \tag{4}$$

The dynamic matrix M and the column vector y are: $M=F+G$ and $y=yc+yt$. The elements of F, G and yc, yt are simple combinations of certain BEPs [15]. The VLBI observed nutations were paired into prograde and retrograde motion ($\eta(\sigma)$) and each one has a real and imagery part. In our case, we have obtained nutation series based on fitting the VLBI observations. But we used nine of them to infer the BEPs, a full set of inversion using all lunisolar terms is undergoing tests. Therefore, in the simplified case, the size of our covariance matrix is $37(4\times 9+1)$, the last element represents the correction to the precession rate. In addition, another matrix (***A***) is built using the partial of $\partial O_\alpha/\partial B_i$, where O_α is observed nutation component and B_i is selected BEPs, which could be the initial values of certain parameters coming from PREM [16] and MHB2000. The corrections to the processing rate were the secular trends found in changes of longitude and it is substituted to the covariance matrix. After each trial the solutions was substituted into the transfer function:

$$T(\sigma; e|e_{\mathrm{R}}) = (1-\sigma/e_{\mathrm{R}})\left[M^{-1}(\sigma)y(\sigma)\right] \tag{5}$$

where σ, eigenfrequency, e, the dynamic ellipticity of the real Earth, e_{R}, the dynamic ellipticity of a rigid Earth. The advantages of the transfer function treatment are that indirect contribution (atmosphere and ocean loading) could be taken into account separately,

Table 4

Estimated BEPs and associated 95% confidence intervals obtained by LSQ fitting of nutation amplitudes (a,$a1$) and Bayesian inversion of the VLBI time series (b,$b1$).

BEPs	MHB2000(1979–1999)^a	1979–2015^{a1}	1979-2010^b	1979–2015^{b1}
$10^3 \times e$	3.2845479 ± 12	3.2845474 ± 2	3.2845481 ± 7	3.2845474 ± 8
$10^3 \times (e_f + ReK^{\text{CMB}})$	2.6681 ± 20	2.6752 ± 15	2.6753 ± 7	2.6760 ± 8
$10^3 \times ImK^{\text{CMB}}$	−0.0185 ± 14	−0.0186 ± 5	−0.0178 ± 4	−0.0188 ± 5
$10^3 \times ReK^{\text{ICB}}$	1.11 ± 10	0.98 ± 6	1.01 ± 3	1.01 ± 4
$10^3 \times ImK^{\text{ICB}}$	−0.78 ± 13	−0.87 ± 22	−1.09 ± 4	−1.11 ± 4

following the same mechanisms of the solid nutations approach [3,17], except for the non-linear terms that are neglected here. At the current stage of the investigation, we are more focused on the BEPs solution with an extended VLBI observed nutation series other than the nonlinear geophysical contributions (loading, anelasticity, CMB topography effects etc.). We leave all those effects which will be further discussed in detail in the following work. Here we kept all the same assumptions than what has been made in the works of MHB2000. By fitting the observations to this theory, we obtained a series of the BEPs, they are:

- Correction to the procession rate;
- Coupling constants at CMB: K^{CMB} (real and imaginary parts);
- Coupling constants at ICB: K^{ICB};
- Dynamical ellipticity of the core: e_f;
- Compliance parameters of the whole earth K;
- Compliance parameters of the fluid core γ.

The processing rate is 5038.482±0.011”/cy (MHB2000) and we found a value 5038.480±0.021“/cy. The analysis yields a mean of 5038.480±0.020”/cy from two models [18]. The different analysis of precession rate is in an agreement within their individual uncertainties assessments using available measurements. However, for a longer period prediction of Earth's precession prediction, the IAU2006 precession model [18] are recommended which has included high order corrections which were ignored in the MHB2000 theory.

The dynamic ellipticity of the out core (e_f) plus the real part of coupling constant at CMB, the imagery part of the coupling constant at CMB and the real part of the coupling constant at ICB are consistent with all determinations. But the imagery part of the coupling constant at ICB is different from the values estimated following the LSQ method and the BI approaches (Table 4). The diagnoses and interpretation of the discrepancy can be found from (Dehant, same issue). Based on the solutions of Liouville equations for an elastic Earth with a liquid core [19,20] and this issue, we could infer the eigenperiod of the FCN (Free Core Nutation), which is observable in the nutations residuals w.r.t. MHB2000, but the FICN (Free Inner Core Nutation) remains undetectable in the measurements. Taking into accounts the bias in the imagery part of the ICB coupling constant, the FICN's period is $P_{\text{ficn}} = 952.1 \pm 53.9$ days and quality factor is $Q_{\text{ficn}} = 443.0 \pm 92.5$. In the case of FCN, it is determined more robustly than FICN. The eigenperiod is $P_{\text{fcn}} = -429.5 \pm 0.7$ days with the quality factor $Q_{\text{fcn}} = 18700 \pm 272$. The amplitude evolution of FCN being of particular interest to the community [21,22], but the origin of long-period modulation need to be consistently studied.

4. Conclusion and discussion

We have analyzed IERS-EOP-C04 VLBI nutation residuals w.r.t. IAU1980 in order to estimate the Basic Earth's Parameters based on the MHB2000 theory. We have used two different approaches. In the first approach BEPs have been estimated in the frequency domain by fitting an updated nutation series with an LSQ method while in the second approach BEPs have been estimated directly from the VLBI time series with a Bayesian Inversion. The computation of the WRMS indicate a better fitting to the observation using our fitted nutation series. Here we provided the terms which have been listed in the MHB2000. The complete set of series will be released after be tested against other more EOP solutions.

The precession rates given by different studies are consistent within their uncertainties using available VLBI observations. These three results suggest a dynamic ellipticity of whole Earth at the range between 0.003284543 and 0.003284548.

Some of the BEPs are suitable to study the Earth's interior, for instance, the flattening of the core the strength of the magnetic field and viscosity at the depth of Earth's core etc [23]. Although the preliminary result show that most of the BEPs are in an agreement with the MHB2000 model. But it is still worth to apply LSQ to the full set of nutations terms, through this way, it will better determine the contribution from diurnal band nutations, meanwhile repeating the BI analysis to multiple extended VLBI global EOP solutions. Furthermore, the loading contribution (atmosphere and ocean) derived from recent analysis [24] will be introduced into the next step study.

As FCN is remaining unpredictable in the long term which obstacles the way to reduce nutations residuals w.r.t. MHB2000 to model it [25]. But the long term trends carried by FCN amplitude changes decomposed into real and imagery parts are remaining unknown due to the absence of a robust excitation mechanism. A suitable excitation model should be able to be employed to model the complete process, in turn after a successful implementation, it will ultimate reduce the nutation residuals. While the FICN is remaining undetectable at the current level of measurement accuracy. It relays on the solved Liouville equations to find its eigenperiod and quality factor.

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