Astrometric bias due to overlapping image profiles in the focal plane and its removal in the positions of near-Earth asteroids

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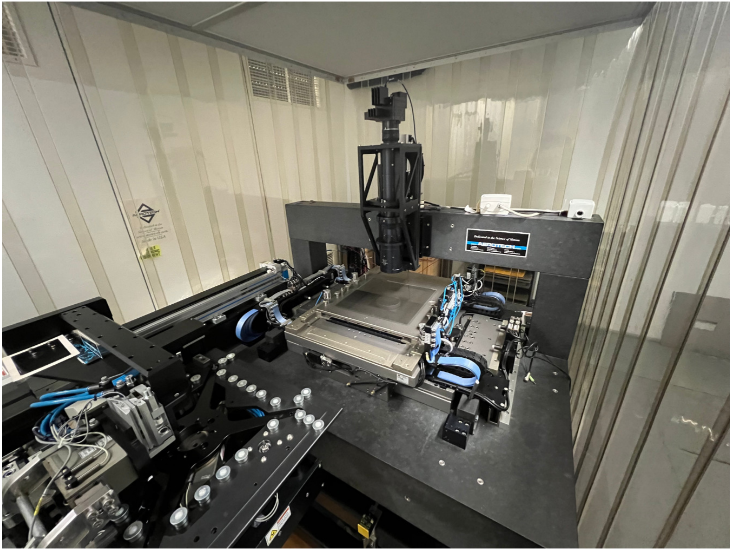
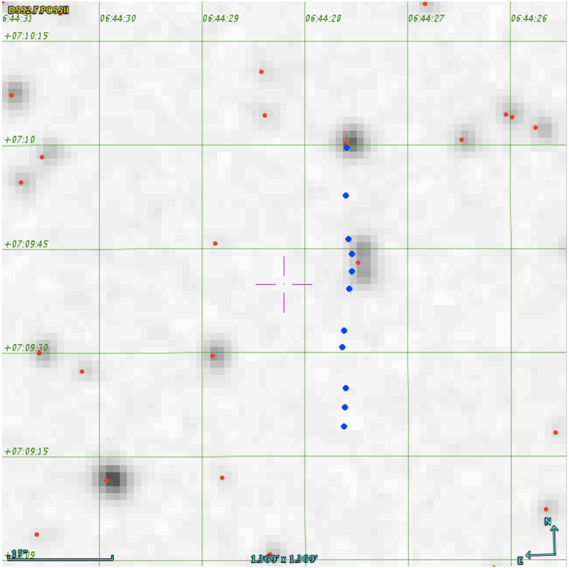
Overview

1. Introduction
2. Analytic solution
3. Analysis of $(O - C)$
4. Discussion and problems
5. Conclusions

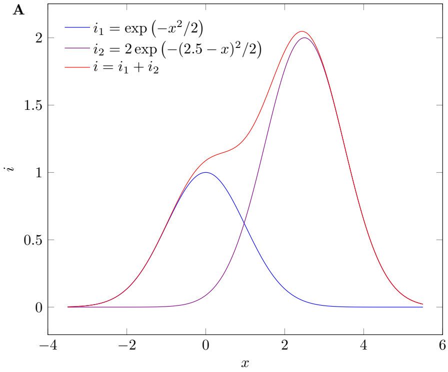
Hi-resolution digitisation using DAMIAN machine at ROB

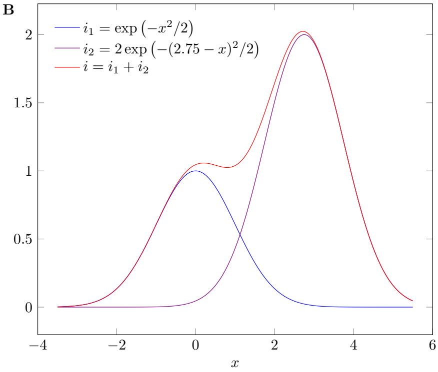


Measurements of (99942) Apophis



Blending Gaussian images



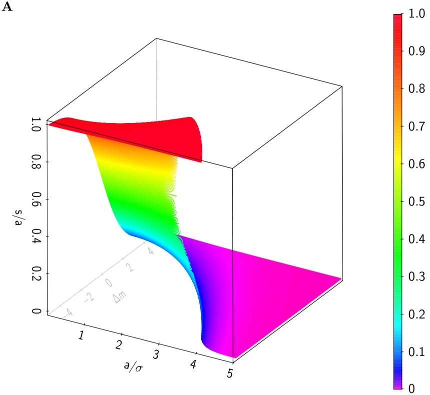


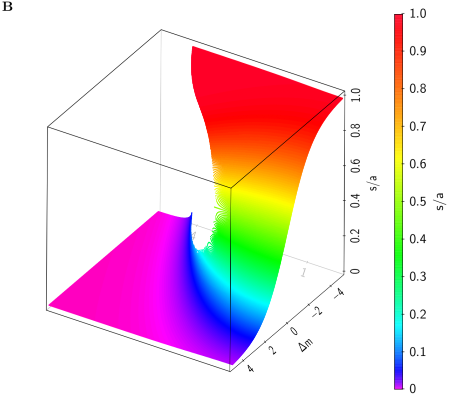
Known bias approximations for **unresolved** case

References	Assumptions	Bias approximation	Error term
Ross, F. (1921) *ApJ* 53, 349	Calculated shift of geometric centroids instead shifts of peaks; PSF was assumed $I \propto \exp(-kr)$	Zero bias if the peaks are used instead the geometric centroids.	
van de Kamp, P. (1937) *AJ* 46, 36	Photocenter position is the weighted mean of the two components; the weights are assigned proportional to the luminosities of the components	$s^{(0)} = \frac{L_2 a}{L_1+L_2} = \frac{a}{1+10^{0.4\Delta m}}$	$O((a/\sigma)^0)$
Hall, R.G., Jr. (1951) *AJ* 55, 215	Discrepancies are discovered experimentally with increasing Δm	Same as before	
Vieira-Martins, R. et al. (2006) *Notes scien. et tech. de l'Inst. de méc. cél.* S087, 51; Assafin, M. et al. (2013) *MNRAS* 430, 2797	PSF is a circular Gaussian; **zero-order** approximation to the maximum was found	$\tilde{s} = \frac{k\sigma^2 a}{\sigma^2 \exp\left(\frac{a^2}{2\sigma^2}\right)+k\left(\sigma^2-a^2\right)}$ *	$O((a/\sigma)^2)$
Benedetti-Rossi, G. et al. (2014) *A&A* 570, A86	Same as before	$\frac{s^{(4)}}{a} = \tilde{s} - \frac{\tilde{s}-k(1-\tilde{s})\exp\left(\frac{a^2}{\sigma^2}\left(\tilde{s}-\frac{1}{2}\right)\right)}{1+k\left(\frac{a^2}{\sigma^2}(\tilde{s}-1)+1\right)\exp\left(\frac{a^2}{\sigma^2}\left(\tilde{s}-\frac{1}{2}\right)\right)}$	$O((a/\sigma)^6)$

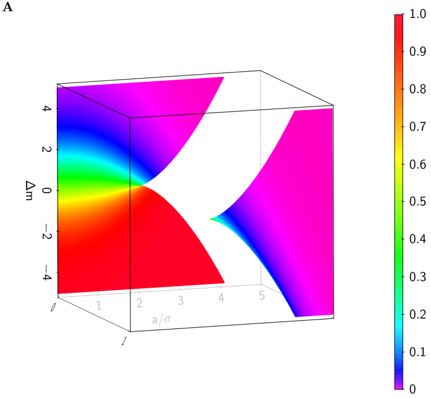
* Using $k = 10^{-0.4\Delta m}$, one can find $\tilde{s} = \frac{a}{1-\frac{a^2}{\sigma^2}+10^{0.4\Delta m}\exp\frac{a^2}{2\sigma^2}} \to \frac{a}{1+10^{0.4\Delta m}}$ if $\frac{a}{\sigma} \to 0$.

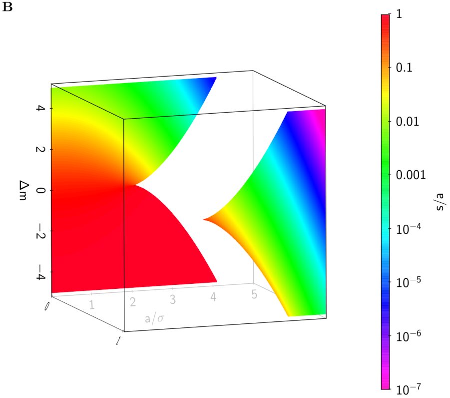
Exact solution $s/a = \frac{s(a/\sigma,\Delta m)}{a}$ for Gaussian images





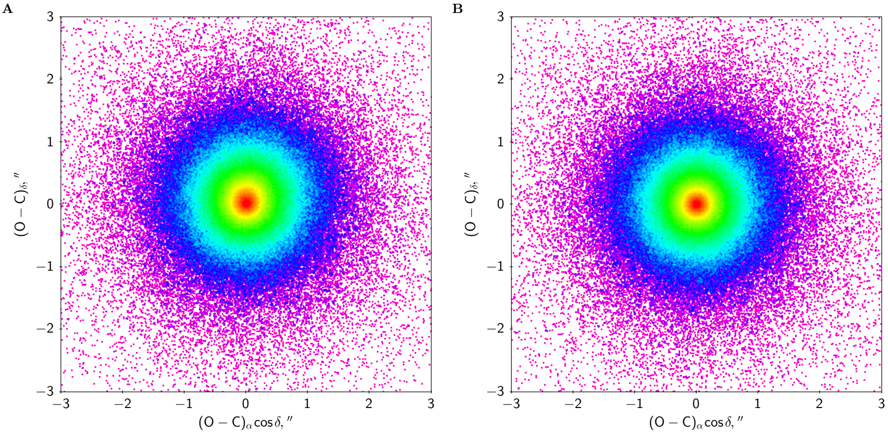
Exact solution $s/a = \frac{s(a/\sigma,\Delta m)}{a}$ for Gaussian images





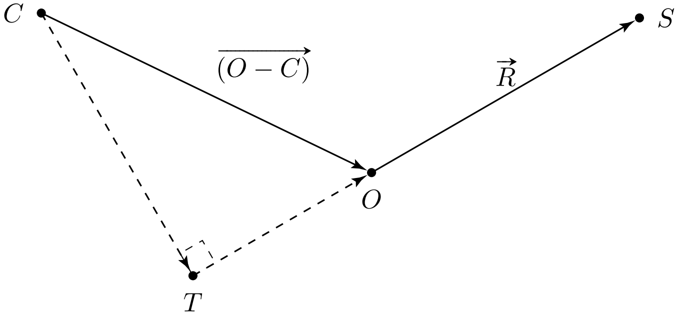
Positions suspected for bias presence

Found 456,416 positions for 2,967 numbered and 21,723 unnumbered NEAs measured within 9″ of Gaia EDR3 stars at 1241 observatories.



Before and after the catalog correction was applied, Eggl, S. et al. (2020) *Icarus* 339, 113596.

Inverse problem for finding the bias $s\,(a, \sigma, \Delta m)$



If $\overrightarrow{CO} = \vec{i}(O - C)_\alpha \cos\delta + \vec{j}(O - C)_\delta$ then a nonlinear equation is

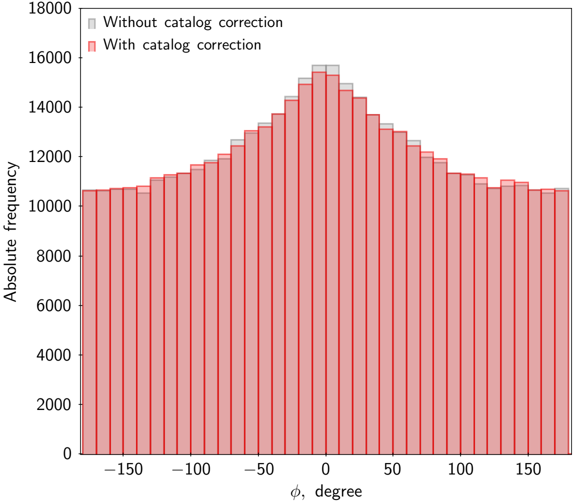
$$\overrightarrow{CO} \cdot \vec{R} = Rs\,(R + s, \sigma, \Delta m) \quad \forall \overrightarrow{CO} \cdot \vec{R} > 0.$$

Angle ϕ and its **observed** distribution

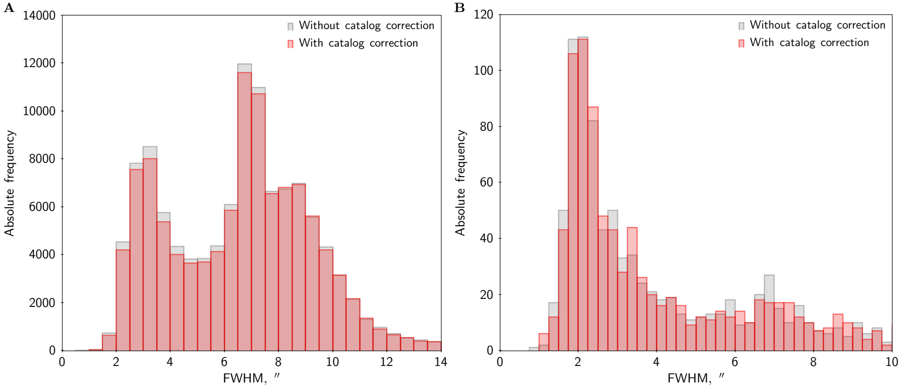
Angular variable:

$$\sin\phi = \frac{\vec{R} \wedge \overrightarrow{CO}}{R\left|\overrightarrow{CO}\right|}$$

$$\cos\phi = \frac{\vec{R} \cdot \overrightarrow{CO}}{R\left|\overrightarrow{CO}\right|}.$$

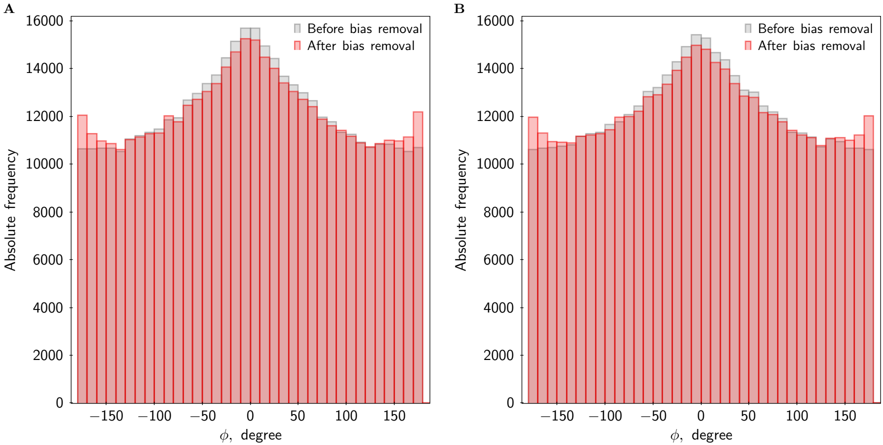


Distribution of the calculated widths σ converted to *FWHM*



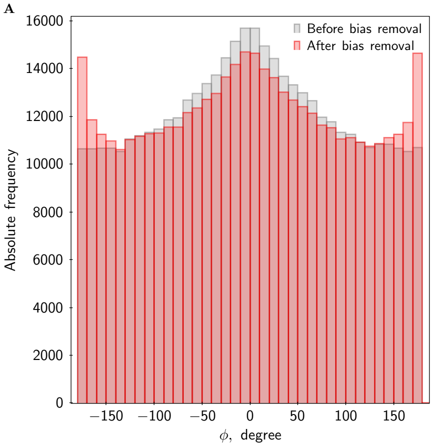
All individual positions of NEAs (A), and the same distribution after being **grouped** by the IAU observatory number using $\tilde{\sigma} = inf\, \sigma_i$ (B).

Distribution (I) of the angle ϕ before and after removal of the bias



Bias removal using $\tilde{\sigma} = inf\,\sigma_i$

Distribution (II) of the angle ϕ before and after removal of the bias



Bias removal using **increased** inf σ_i

Conclusions

- The astrometric bias described **appears due to measurements** (non-coherent accumulating of light) and depends on the image *FWHM* at the specific telescope, its focusing, photometric band, atmospheric conditions, etc. The image width can be determined by either direct measurements or fitting image profiles and is recommended to be reported to the IAU Minor Planet Center using the ADES format.
- Astrometry of both resolved and unresolved objects can be corrected due to the objects nearby.
- Astrometric positions of asteroids measured close to the stars **are likely biased**. We recommend these measurements be down-weighted or eliminated from the orbit fitting process.
- The bias can be eliminated using image **modeling for a group of objects**.

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