

Individual kappa calculation for BELSHAKE, site kappa and kappa gradient estimation across crustal domains in Belgium

Abstract

This study explores the high-frequency attenuation parameter, κ (kappa), an important parameter in seismic hazard assessment. In this study κ was calculated using the classic definition by Anderson and Hough, which is based on the slope of the acceleration Fourier amplitude spectrum.

To refine κ , we:

- Compared κ values with t^* (t-star) values;
- Filtered induced and shallow seismic events (<8 km);
- Extrapolated site-specific κ_0 to zero epicentral distance to isolate site effects;
- Validated results with global V_{330} expectations.

To further investigate κ , three methods for estimating κ gradients and site-specific κ_0 were evaluated:

- Free Kappa Gradient Method,
- Joint Kappa Gradient Method,
- Mixed-Effect Method.

Each method offers unique advantages:

- Free Kappa Gradient Method being computationally efficient but sensitive to data quality;
- the Joint Kappa Gradient Method integrating multi-source data but constrained by optimization ranges;
- the Mixed-Effect Method providing robust statistical reliability for complex datasets.

Performance comparisons across crustal domains using the Intraclass Correlation Coefficient highlight the trade-offs in simplicity, robustness, and accuracy among these methods. This comprehensive analysis enhances the understanding of crustal attenuation properties, contributing to improved seismic models and hazard assessments.

Introduction

κ quantifies the decay of the acceleration Fourier amplitude spectrum at high frequencies, reflecting both site-specific attenuation (κ_0) and regional path effects (κ_r). This parameter is applied in:

- Ground-motion simulations.
- Host-to-target adjustments of ground-motion models (GMMs).
- Site-specific analyses for critical infrastructure, where accurate attenuation parameters are essential.

These applications have demonstrated κ 's utility in addressing site effects, regional tectonic variability, and near-surface geological structures. In this study kappa is calculated for all the records in BELSHAKE database. The BELSHAKE project is funded by BELSPO (Belgian Science Policy) under the BRAIN-be 2.0 Program and aims to:

- Build a database of earthquake ground motion for Belgium and adjacent areas, including:
 - Systematic inventory of digital records of local and regional earthquakes by stations of the Belgian network;
 - Implementation of a semi-automatic workflow to uniformly process seismograms and determine ground-motion parameters following international standards;
 - Collection of ground-motion parameters and associated metadata in a database.
- Develop the capacity to model ground motion due to earthquakes, including:
 - Characterization of seismic attenuation (kappa) in the main crustal domains in Belgium;
 - Evaluation of existing ground-motion models (GMMs);
 - Development of a stochastic ground-motion model for Belgium or calibration of a regionally adaptable ("backbone") GMM.

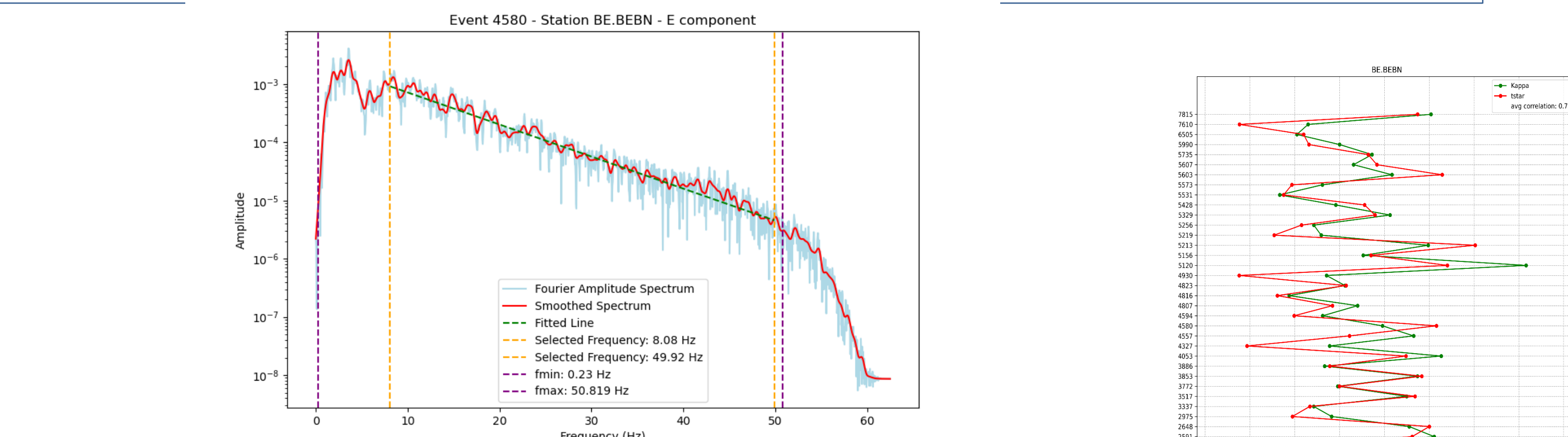


Figure 1. Fourier amplitude spectrum of the S-window for E component of a record by BE.BEBN station related to the event_ID=4580 in BELSHAKE database, with $M_L=4.3$, date:2011-09-08, epicenter location:GOCH (DE).

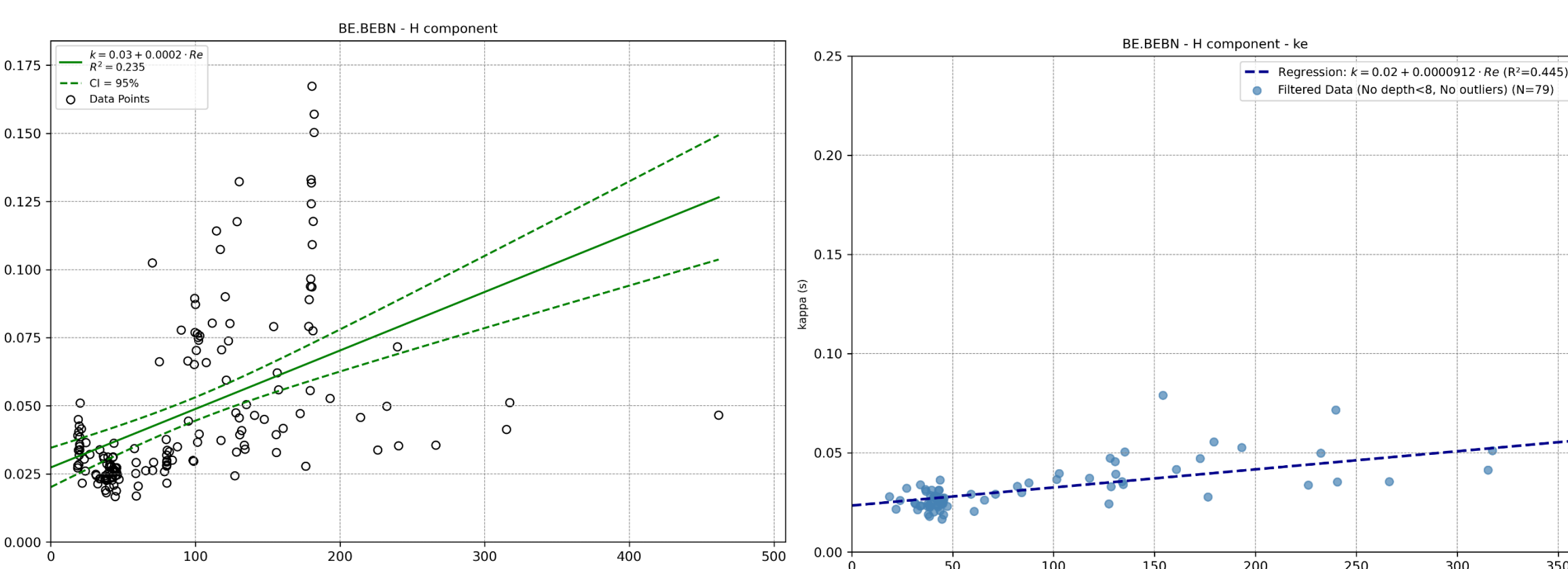


Figure 2. Kappa versus epicentral distance for BE.BEBN station. Left: All κ values, Right: Filtered κ values after removing induced events and shallow records.

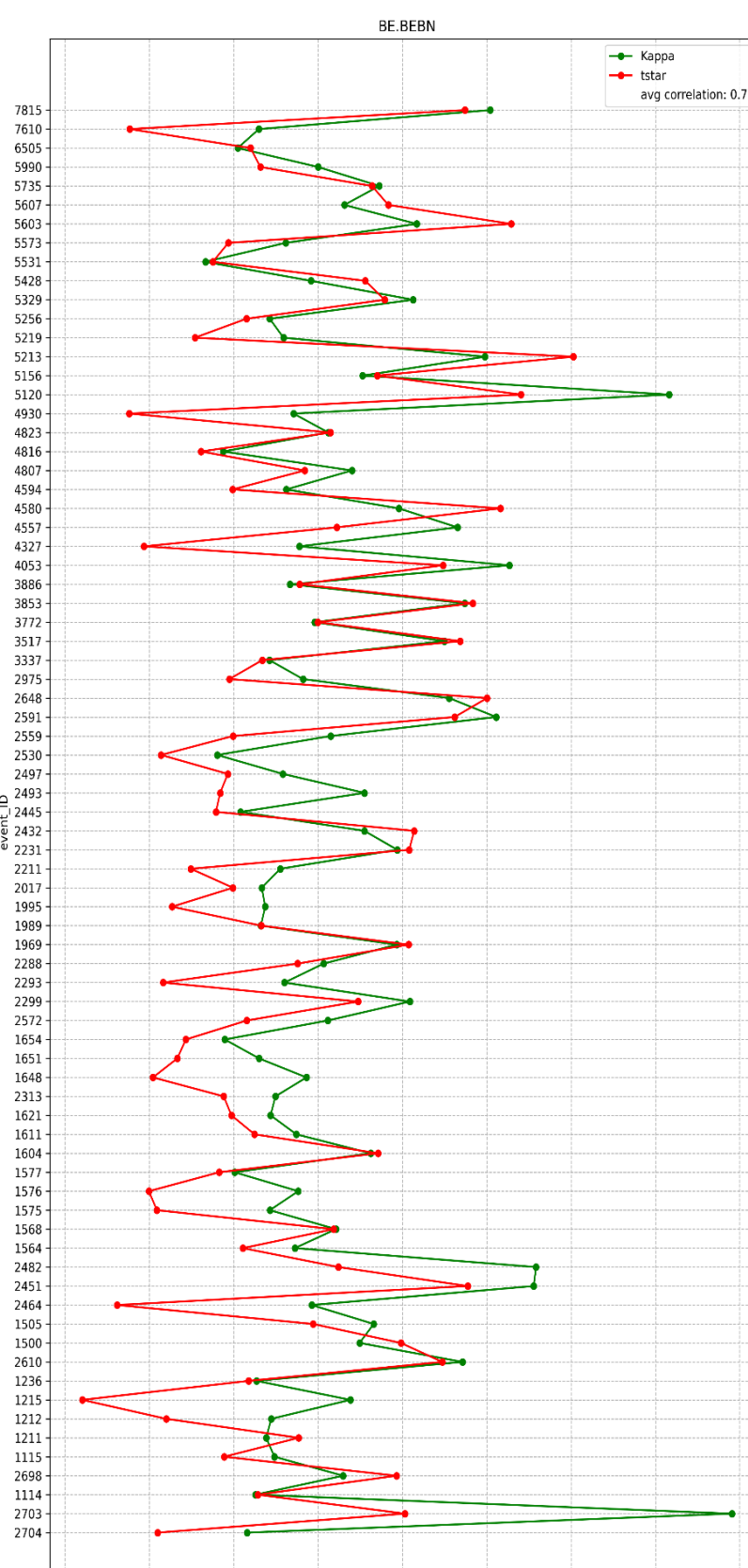


Figure 3. correlation between kappa and t-star value, validating individual kappa calculation

Methods

For each record

- The minimum (f_1) and maximum (f_2) frequencies of the linear portion of the Fourier amplitude spectrum were visually selected for the vertical (Z), east (E), and north (N) components.
- These frequencies were constrained by a pre-established frequency range ($f_{min}-f_{max}$) for the BELSHAKE dataset, ensuring a minimum SNR of 3. Resonance peaks due to site effects were excluded.
- The linear portion of the smoothed spectrum was identified, and the slope was calculated using ordinary least squares (OLS).
- The κ value for each component was obtained by dividing this slope by π .
- For the horizontal (H) component the arithmetic average of the κ values from the east (E) and north (N) components was used.

Figure 1 illustrates the Fourier amplitude spectrum of the S-wave window for the vertical component of a record in BELSHAKE. This approach ensures accurate and consistent κ values by visually inspecting the Fourier spectra, applying strict frequency selection criteria, and avoiding noise and site resonance effects.

After calculating κ values for each station,

- We assessed their relationship with epicentral distance, observing a general trend of increasing κ with increasing R_{epi} (Figure 2, left). While most stations followed this trend, certain anomalies were observed as vertical clusters in the κ versus R_{epi} plots. These clusters were linked to induced seismic events and shallow events (depth < 8 km). To resolve this, a filtering process was applied (Figure 2, right).

- We also examined the correlation of kappa with t^* (t-star) values for each station together with moment magnitude. t^* is defined as the travel time divided by Q . Figure 3 demonstrates a strong correlation of 0.75 between κ and t^* for the BE.BEBN station, a trend that is consistent across other stations as well, further validating the methodology.

SITE KAPPA AND KAPPA GRADIENT ESTIMATION

The next part of study investigate three methods for estimating kappa gradients of different crustal domains and site kappa values:

- Free Kappa Gradient Method,
- Joint Kappa Gradient Method, and the
- Mixed-Effect Method.

These parameters (kappa gradient and site kappa) are critical for understanding the spatial variability of crustal properties and improving seismic ground-motion models, and are calculated using the following formula:

$$\kappa = \kappa_0 + \kappa_r \cdot r$$

This formula distinguishes site kappa (κ_0) from kappa gradient (κ_r), where r is the distance from source to site. To isolate site-specific effects, the site-specific κ_0 was determined by extrapolating κ values to zero epicentral distance. Figure 5 demonstrates the estimation of κ_0 for BE.BEBN station as the intercept, and κ_r for ARD crustal domain as the slope of the linear fits.

- The derived κ_0 values were compared against global expectations based on each station's V_{330} previously computed and stored in the database. The κ_0 values showed good agreement with global values.

- The kappa gradient was derived for each crustal domain. Belgium is categorized into five crustal domains shown in Figure 4;

- ABM (Anglo-Brabant Massif),
- ARD (Ardenne),
- RVG (Roer Valley Graben),
- HAIN (Hainaut) and
- WAZ (Weald-Artois Zone).

- The Free Kappa Gradient Method is straightforward, relying on station-specific data for calculations. While efficient, it can produce unreliable estimates when data quality or station coverage is limited.
- The Joint Kappa Gradient Method integrates data from multiple sources and uses optimization techniques. Although more robust, it requires predefined model parameter ranges, which can reduce flexibility in some scenarios.
- The Mixed-Effect Method applies a statistical framework that accounts for fixed effects (attributed to each crustal domain) and random effects (attributed to individual stations), making it ideal for datasets with uneven station coverage or high variability. Results of these three methods are shown in Figure 6.

The performance of these methods was compared across different crustal domains using the Intraclass Correlation Coefficient (ICC) to assess consistency and reliability. Figure 7 shows the result of this comparison.

- For estimating site kappa, the Mixed-Effect Method aligns well with the Free Kappa Gradient Method, but only for stations with sufficient amount of data.

- For estimating kappa gradient estimation across crustal domains, the Mixed-Effect Method shows a stronger correlation with the Joint Kappa Gradient Method, highlighting its effectiveness in capturing regional attenuation properties.

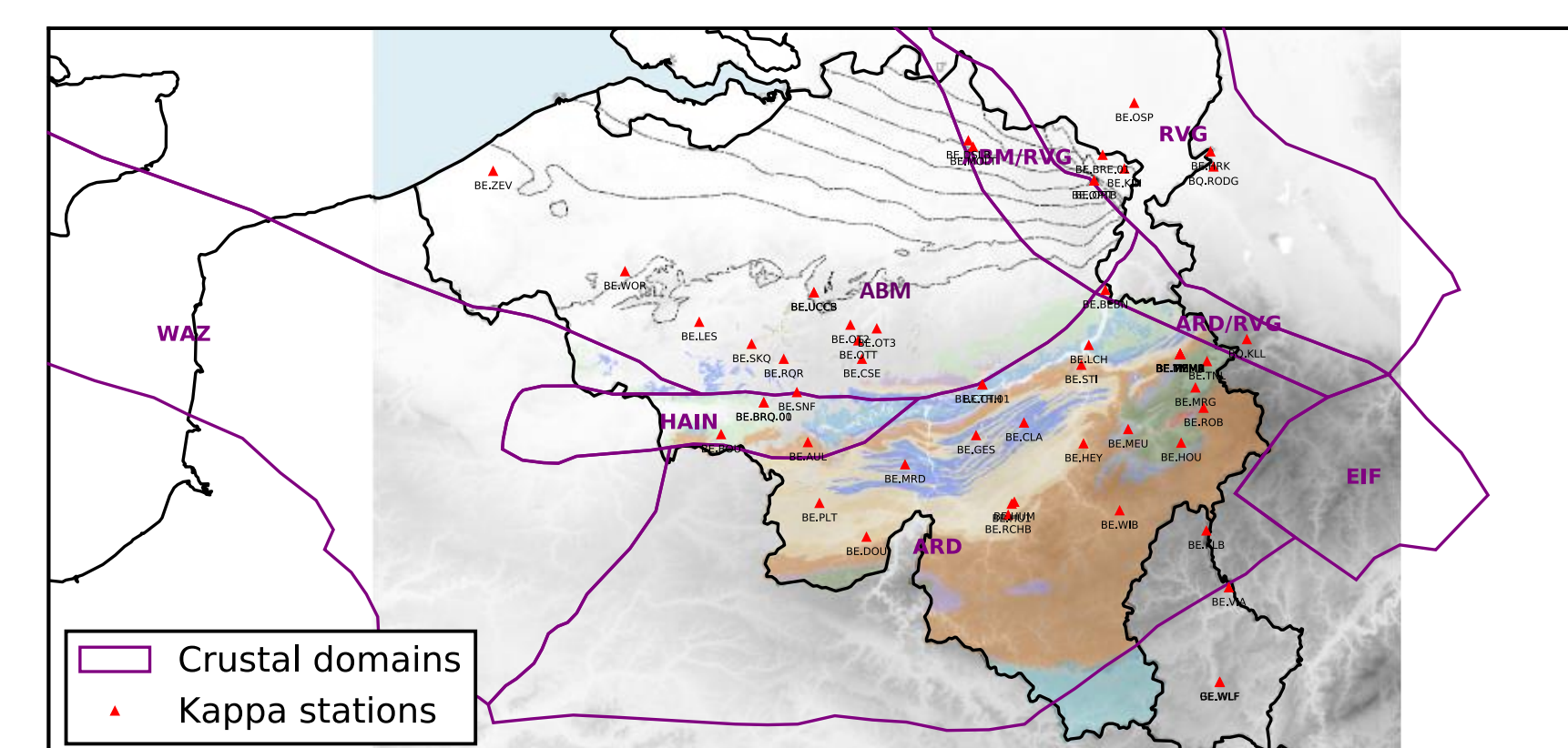


Figure 4. crustal domains and stations in Belgium

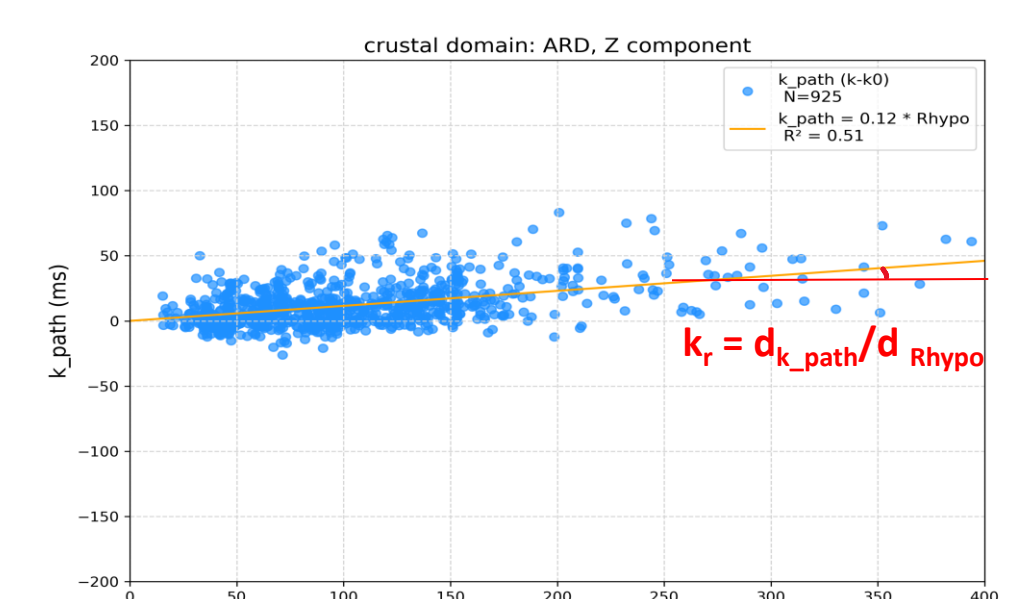
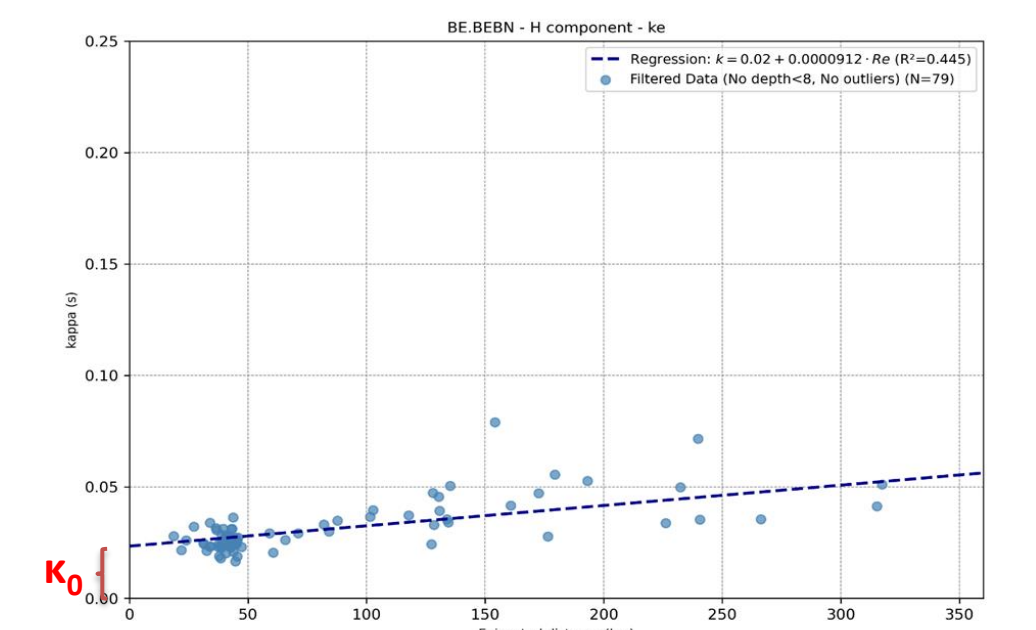
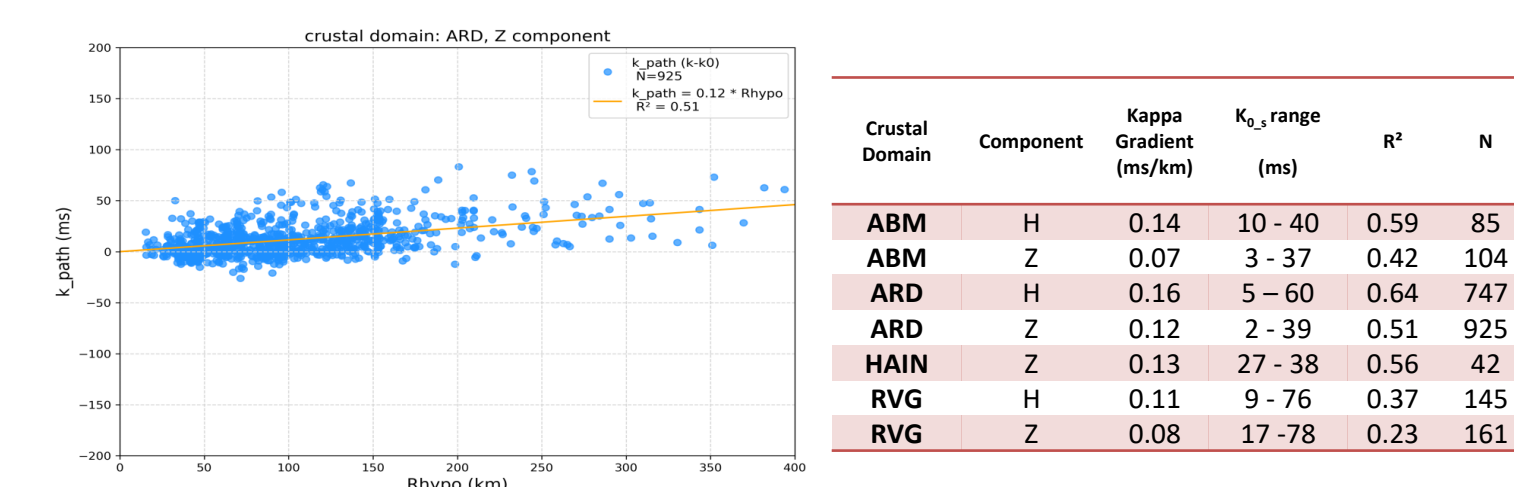
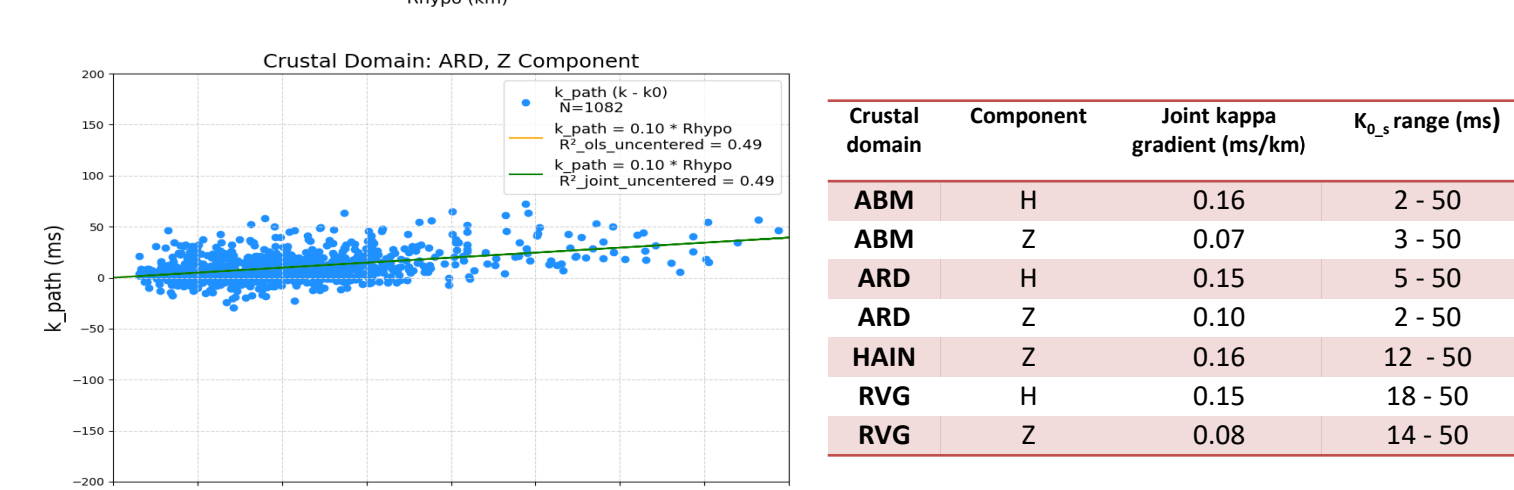


Figure 5. site kappa estimation for BE.BEBN and kappa gradient estimation for ARD crustal domain



Crustal domain	Component	Kappa Gradient (ms/km)	κ_0 range (ms)	R ²	N
ABM	H	0.14	10-40	0.59	85
ABM	Z	0.07	3-37	0.42	104
ARD	H	0.16	5-60	0.64	747
ARD	Z	0.12	2-39	0.51	925
HAIN	Z	0.13	27-38	0.56	42
RVG	H	0.11	9-76	0.37	145
RVG	Z	0.08	17-28	0.23	161

Figure 6. First row: result of Free Kappa Gradient Method second row: result of Joint Kappa Gradient Method Third row: result of Mixed Effect Method

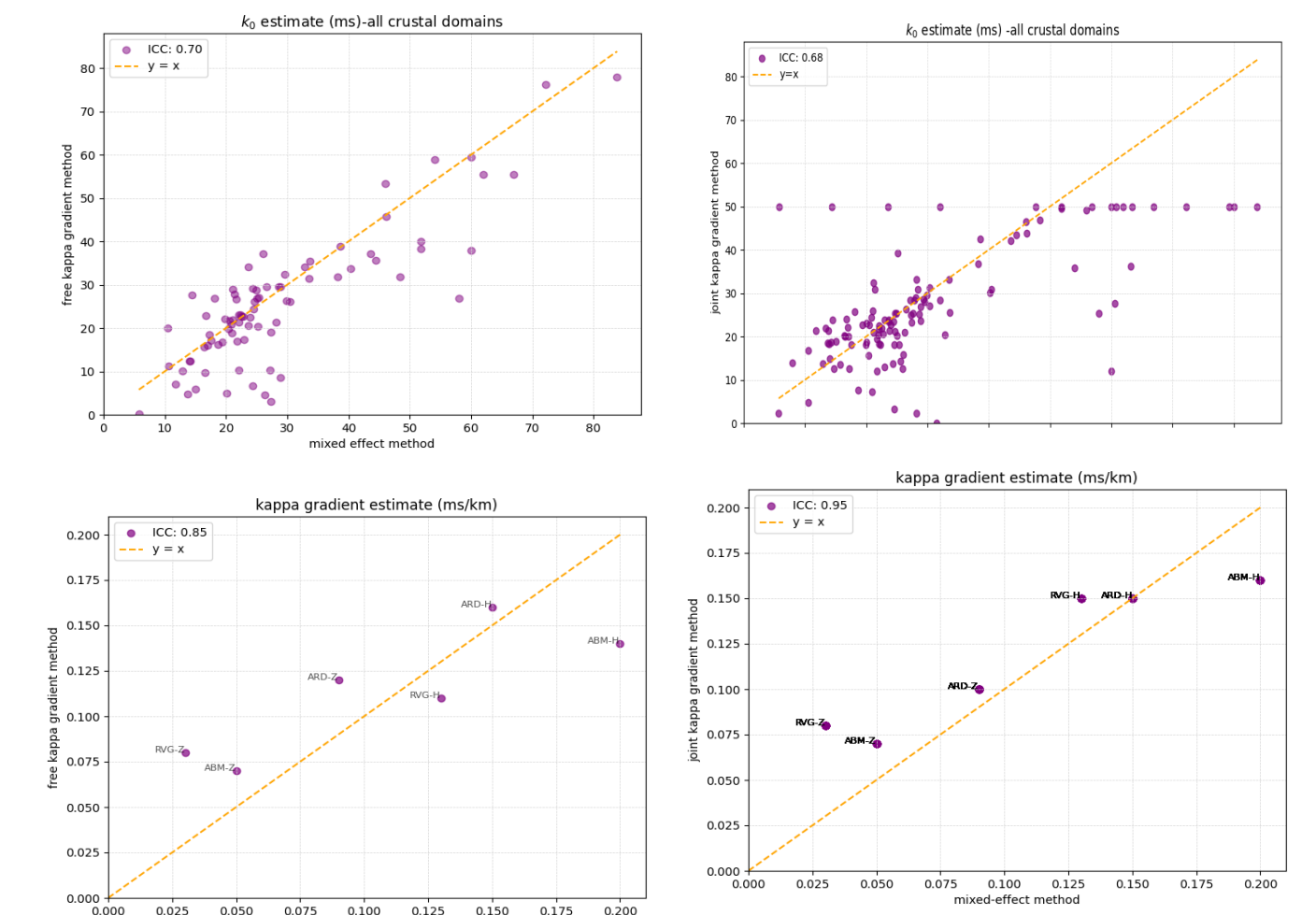


Figure 7. comparing the Mixed Effect Method results with the Free Kappa Gradient Method and the Joint Kappa Gradient Method in estimating κ_0 and kappa gradient.

Conclusions

- The kappa values were computed using the classical method of Anderson and Hough (1984), showing a general increase with epicentral distance. Anomalies from shallow and induced events were filtered out.
- Strong correlations between kappa and t^* confirmed the robustness of the methodology.
- Site-specific κ_0 values were derived and validated against V_{330} data, demonstrating good agreement with global expectations.
- Comparison of three methods for estimation site kappa and kappa gradient of crustal domains show that:
 - While the free kappa gradient method is simple and computationally efficient, it is highly sensitive to data availability.
 - The joint kappa gradient method offers a more sophisticated approach but can suffer from optimization range issues that limit its effectiveness.
 - The mixed-effect method, by contrast, is better equipped to handle the complexities of crustal domain data, making it the most dependable option for estimating kappa gradients of each crustal domain and κ_0 for each station.

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- Olga-Joan Ktenidou, Fabrice Cotton, Norman A. Abrahamson, and John G. Anderson (2014). Taxonomy of κ : A Review of Definitions and Estimation Approaches Targeted to Applications.