The return of tesseral Rossby waves in a rotating sphere due to stable stratification.

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1. INTRODUCTION

Motivation

Rossby waves[a] are a class of inertial waves, waves restored by the Coriolis force in rotating fluid systems, that are characterized by the conservation of absolute vorticity. Understanding how their properties depend on the underlying fluid system is key in using these waves to probe the stellar and planetary systems in which they are observed. To that end much progress has been made in deriving exact solutions and dispersion relations for radially or equatorially trapped[b] Rossby waves. Here we use a global 3D numerical eigenvalue computation and the Boussinesq approximation in a stably stratified sphere or spherical shell to **study Rossby waves that fill the spatial domain and to determine how their frequency and damping depends on the shell width and stratification strength.**

Theory on Rossby waves

In spherical geometry θ, ϕ, the analytic **2D** dispersion relation for hydrodynamic Rossby waves is given by:

$$\frac{\omega}{\Omega} = \frac{-2m}{\ell(\ell+1)} \qquad (1)$$

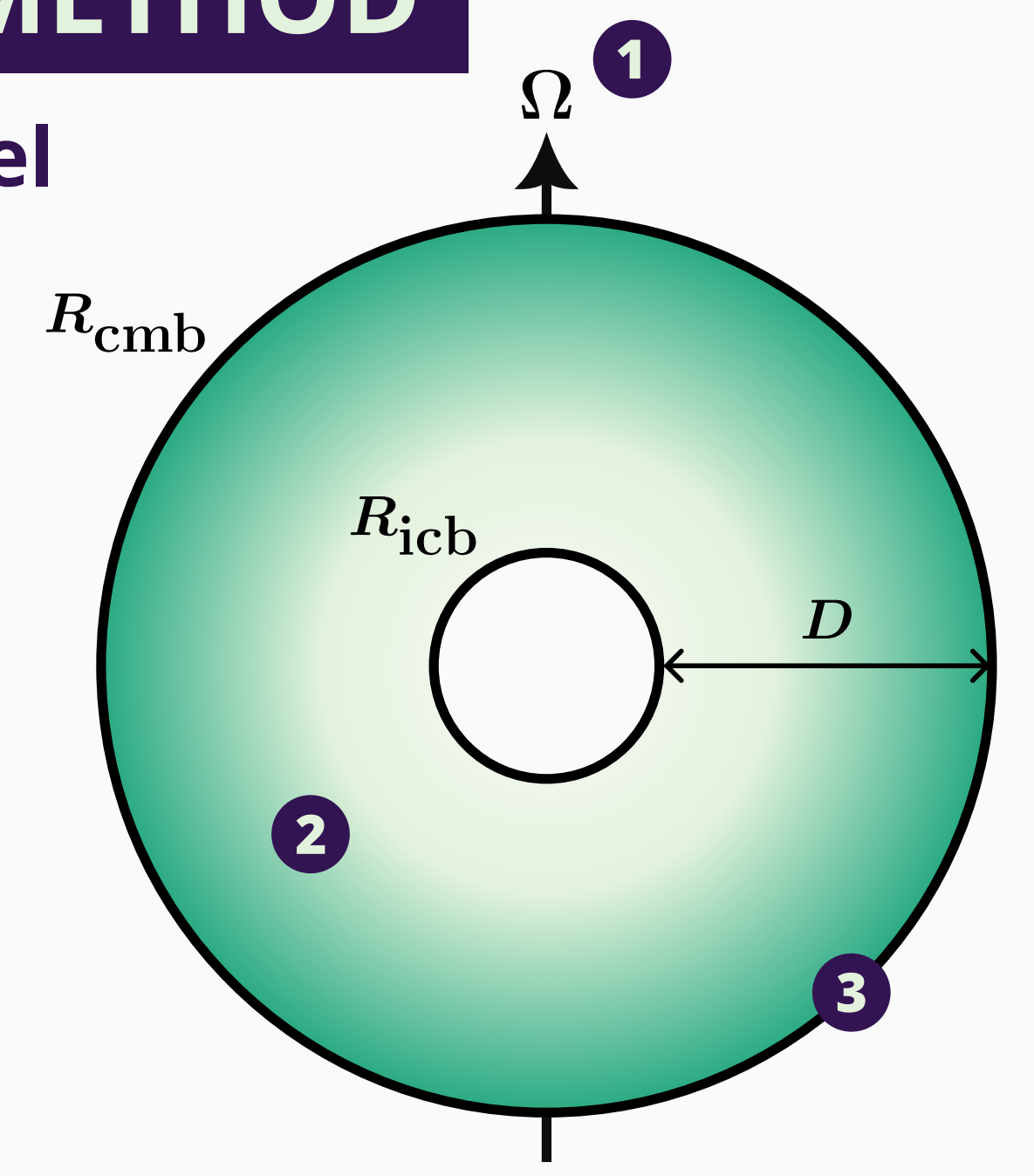
ω angular frequency of the wave;
Ω angular frequency of the rotating system;
m harmonic wave order;
ℓ harmonic wave degree.

$\ell = m$	sectoral Rossby waves
$\ell \neq m$	tesseral Rossby waves

In the main figure the analytical frequencies for the $m = 2$ Rossby waves are indicated with the dashed (sectoral $\ell = 2$) and solid (tesseral $\ell = 2, \dots, 12$) horizontal lines.

2. METHOD

Model



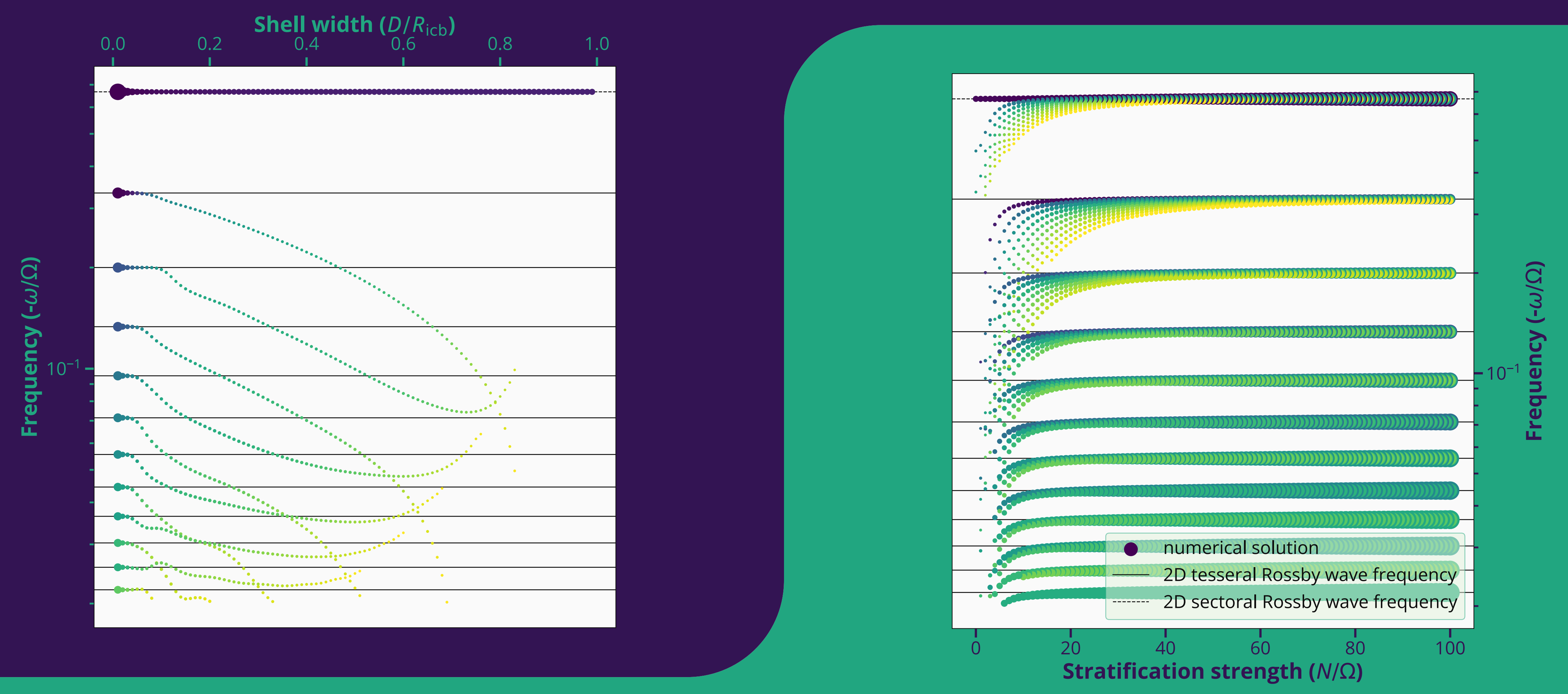
1. Spherical shell with **shell width** D, rotating with constant angular frequency Ω;
2. Linear background thermal stratification whose strength is determined by the value of the **Brunt-Väisälä frequency** N at the outer boundary R_{cmb};
3. Stress-free and constant heat flux conditions on the inner R_{icb} and outer core R_{cmb} boundary

Numerical computation

Starting from the standard linear, non-dimensional **Boussinesq MHD equations** for a viscous, incompressible fluid[c] we use a **fully spectral decomposition** of the velocity and temperature field and solve the resulting algebraic equations with the numerical code Kore.

KORE

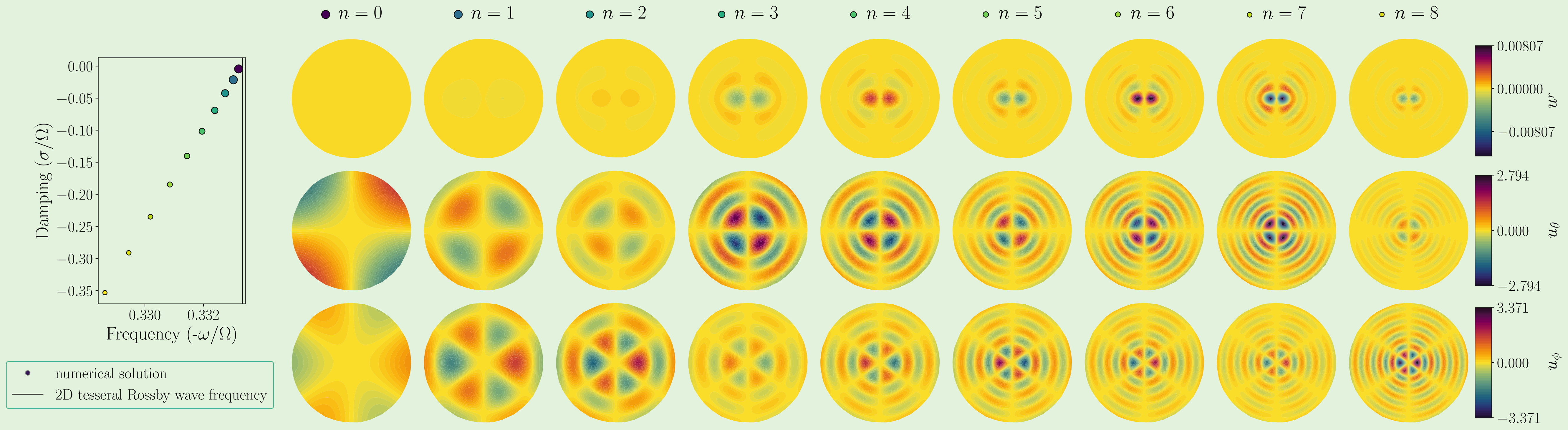
The 2D dispersion relation fails for tesseral Rossby waves when the width of the spherical shell domain is increased.



Strong enough stratification brings the tesseral Rossby waves including several overtones back to the 2D frequency.

Overtones of the tesseral Rossby waves

For sufficient stratification ($N/\Omega \gtrsim 10$) we find multiple Rossby-like waves near each analytical frequency that increase in frequency, damping and radial order, see also the meridonial cuts ($\phi = 0$) of the radial, latitudinal and azimuthal velocity for the tesseral Rossby-like waves with $m = 2$, $\ell \approx 3$ and $N/\Omega = 100$ below.



In the main figure the least-damped Rossby-like waves for $m = 2, \mathrm{Ek} = 0.0003$ and $\mathrm{Pr} = 1$ are shown, varying the shell width for $N/\Omega = 0$ and varying the stratification strength for $D = R_{cmb}$. Each coloured marker is a solution to the wave equations where:

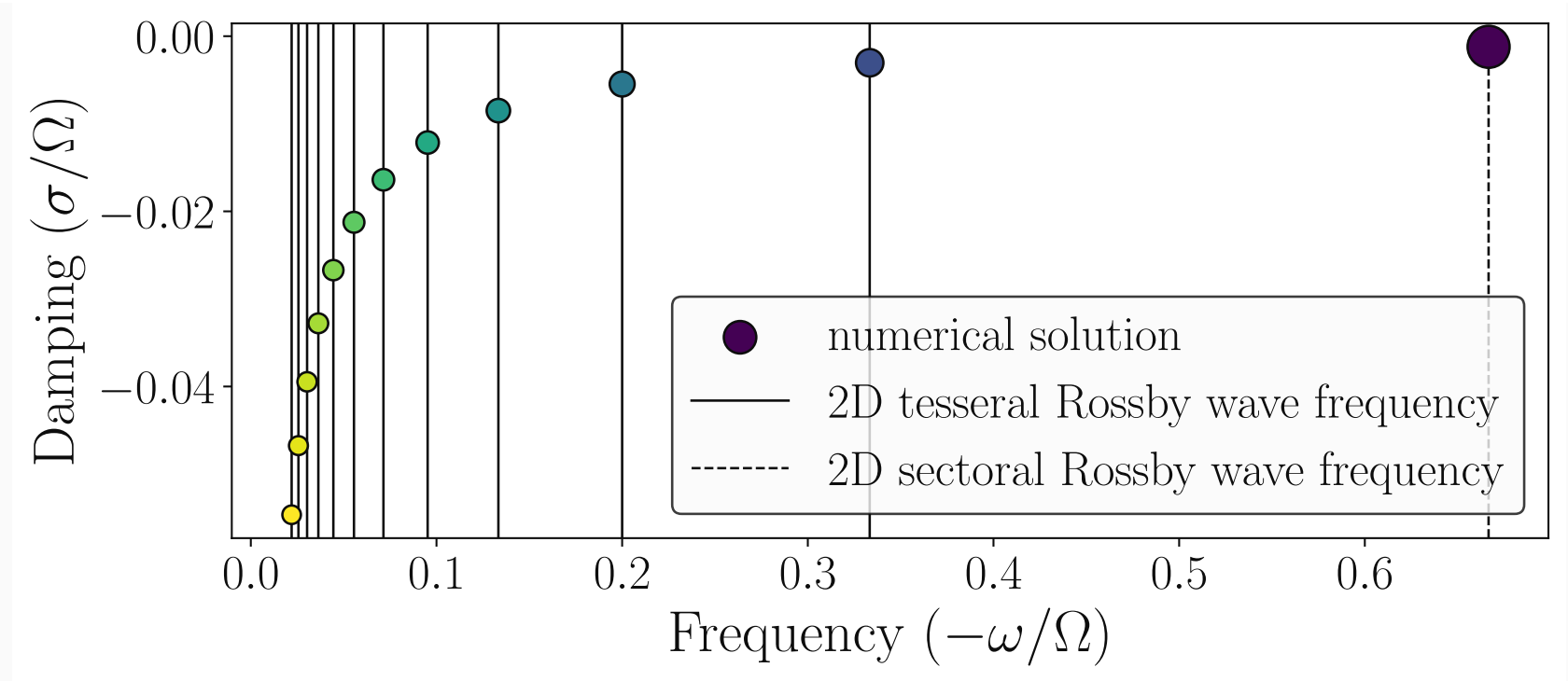
- marker size reflects the toroidal to poloidal kinetic energy ratio
  - large dots correspond to waves with stronger horizontal versus radial motions;
- marker colour reflects the wave damping
  - the darkest dots correspond to the least-damped waves that are easiest to excite.

-4.49e-01 -5.49e-01 -6.70e-01 -8.19e-01
Damping (σ/Ω)

3. CONCLUSION

Thin spherical shell

To a good approximation the Rossby waves in a thin spherical shell with $D/R_{\text{icb}} = 0.01$ follow the 2D dispersion relation. The sectoral wave is the least damped.

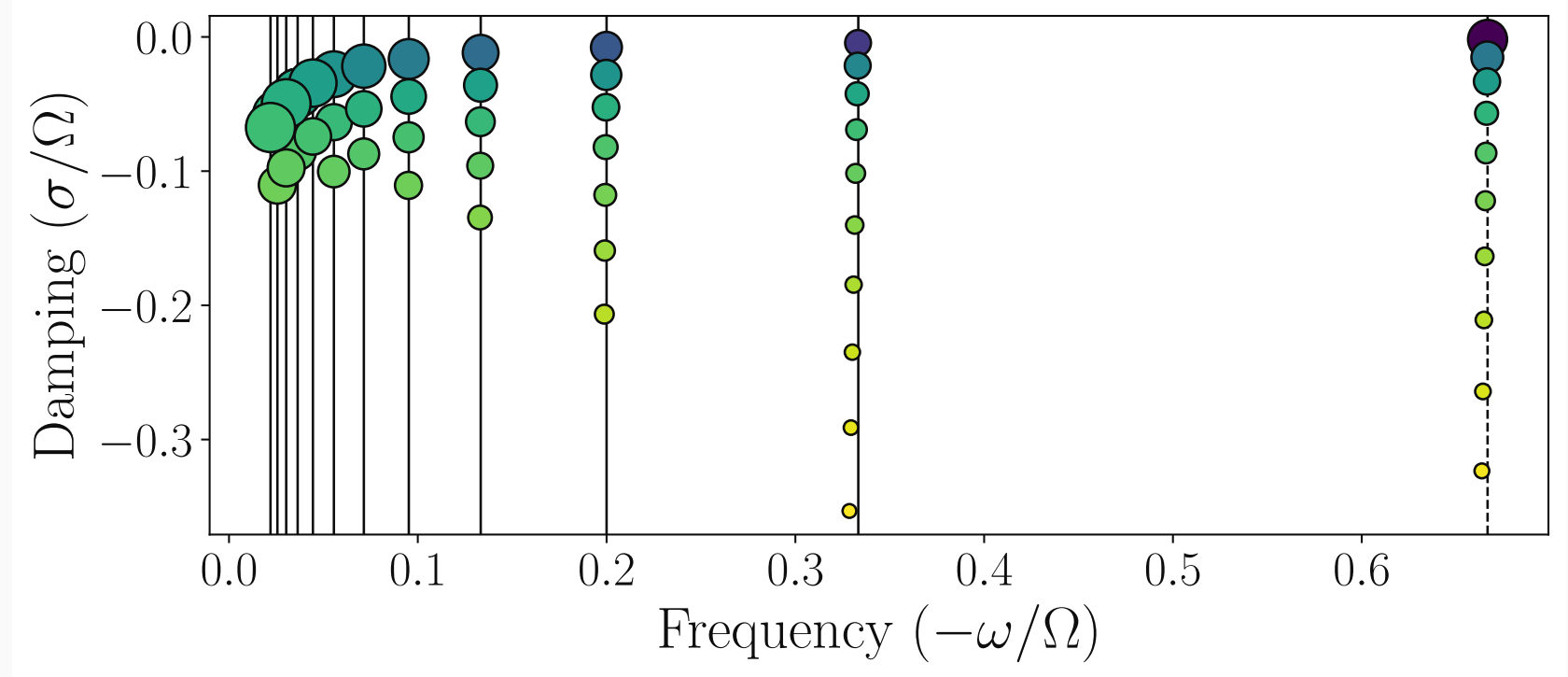


Increasing shell width

As the shell-width increases, the sectoral Rossby wave is the only wave that retains its frequency.

Strong stratification

Stratification strongly surpresses radial motions and effectively replaces the lower boundary so that the 2D dispersion relation remains valid for strong stratification with Brunt-Väisälä frequency $N/\Omega = 100$. In this case the Rossby waves can fill the spherical domain and radial overtones with increasing radial order can be observed.



Decreasing stratification

As the stratification strength decreases, only the sectoral Rossby wave with lowest radial order $n = 0$ remains.

4. DISCUSSION

Future work

- Adopt the anelastic over the Boussinesq approximation to facilitate comparison with the exact solutions for equatorially trapped waves.
- Adopt different stratification profiles to compare with Rossby wave observations in the Sun[d] and stars[e], for example by including a neutrally stratified outer layer to search for possible wave coupling with inertial waves in a convective envelope.

GRACEFUL

Acknowledgements

The research leading to these results has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (GRACEFUL Synergy Grant agreement No 855677). J.R. gratefully acknowledges funding from the "Fonds voor Wetenschappelijk Onderzoek – Vlaanderen" (FWO) Travel Grant V441323N.

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