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The influence of stably stratified layers in the fluid cores of planets

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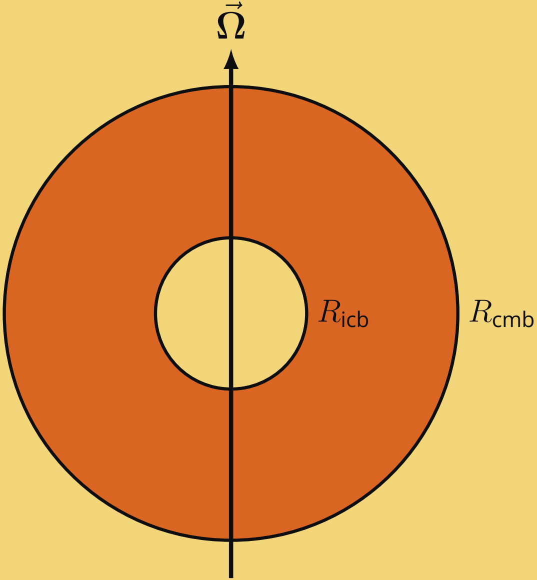
Simple example of the fluid layer in a planet.



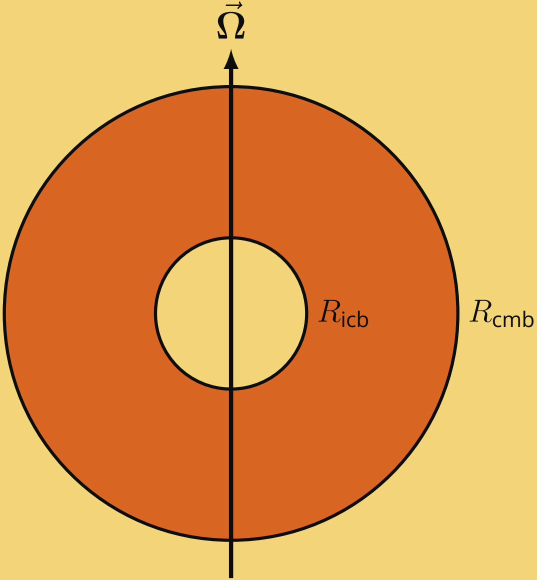
- incompressible, homogeneous fluid

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{u}} = 0 \; ; \tag{1}$$

- no background density gradient;
- no magnetic field;
- solid inner core and mantle rotating with constant angular velocity Ω

$$\vec{\boldsymbol{\Omega}} = \Omega \hat{\mathbf{z}} \; . \tag{2}$$

Simple example of the fluid layer in a planet.



Linear, non-dimensional version of the Navier Stokes equation in the frame rotating with constant angular velocity Ω.

$$\partial_t \vec{\mathbf{u}} + 2\hat{\mathbf{z}} \times \vec{\mathbf{u}} = -\boldsymbol{\nabla} p + \mathrm{Ek}\boldsymbol{\nabla}^2 \vec{\mathbf{u}} \,, \tag{3}$$

with $\mathrm{Ek} = \nu / \Omega R_{cmb}$, the ratio between *viscous* and *Coriolis forces*.

Solutions to this equation are called inertial waves.

Inviscid inertial waves.

The dispersion relation setting Ek $= 0$:

$$\omega = \pm 2\hat{\mathbf{z}} \cdot \hat{\mathbf{k}} \tag{4}$$

- Frequency ω independent of wavelength $|\mathbf{k}|$
- group velocity $\mathbf{v}_g$ is perpendicular to phase velocity $\mathbf{v}_p$

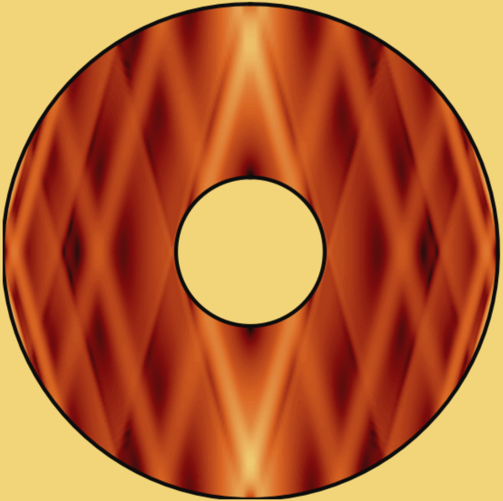
$\Rightarrow$ inertial waves can superpose to form wave packets that propagate with $\mathbf{v}_g$ along straight characteristics of the equation.

Reflection law of inviscid inertial waves.

When inertial waves reflect off solid boundaries the angle $\gamma = \hat{\mathbf{k}} \cdot \hat{\mathbf{z}}$ is conserved.

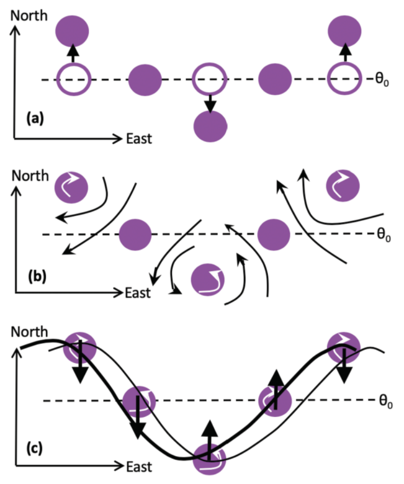
$$\tan \gamma = \pm \left(\frac{4 - \omega^2}{\omega^2} \right)^{-1/2} \quad (5)$$

Viscous inertial waves.



Reintroducing viscosity the (numerical) solutions for inertial waves are characterized by internal shear layers.

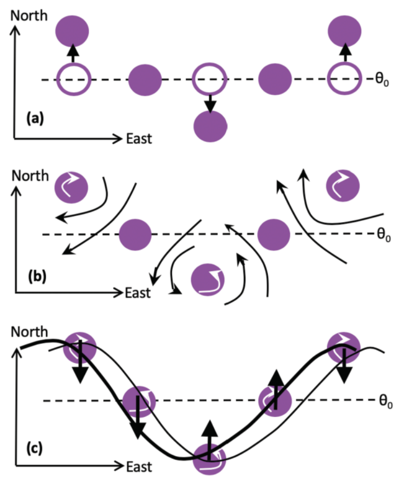
Rossby waves.



Rossby waves are a type of inertial waves characterized by the conservation of absolute vorticity ζ_a.

◄ Fig 1. Illustration of Rossby wave mechanism by Zaqarashvili et al. (2021).

Rossby waves.

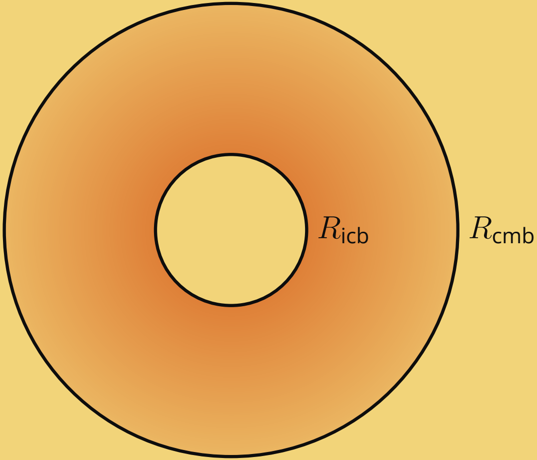


In spherical geometry θ, ϕ the **2D** dispersion relation of Rossby waves is given by:

$$\frac{\omega}{\Omega} = \frac{-2m}{\ell(\ell+1)} \tag{6}$$

where m and ℓ are the harmonic order and degree of the waves.

◄ Fig 1. Illustration of Rossby wave mechanism by Zaqarashvili et al. (2021).



Gravity waves.

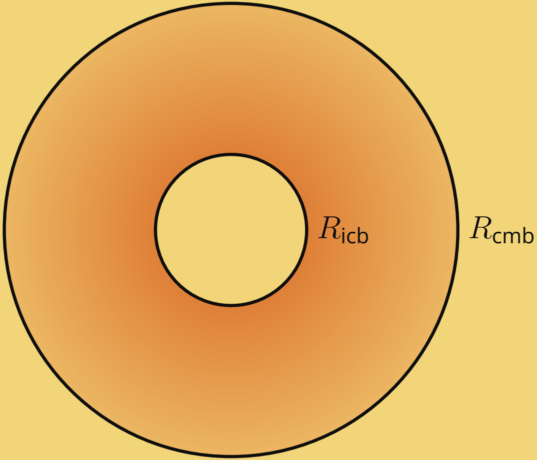
Ignoring rotation and viscosity, but introducing radial stratification in the form of gravity $\hat{\mathbf{g}} = g\hat{\mathbf{r}}$ results in:

$$\partial_t \vec{\mathbf{u}} = -\boldsymbol{\nabla} p - \frac{\rho'}{\rho_0} g\hat{\mathbf{r}} \,, \tag{7}$$

$$\partial_t \rho' = -\vec{\mathbf{u}} \cdot \boldsymbol{\nabla} \rho_0 \tag{8}$$

Solutions to this set of equations are called gravity waves.

Gravity waves.



The dispersion relation for gravity waves:

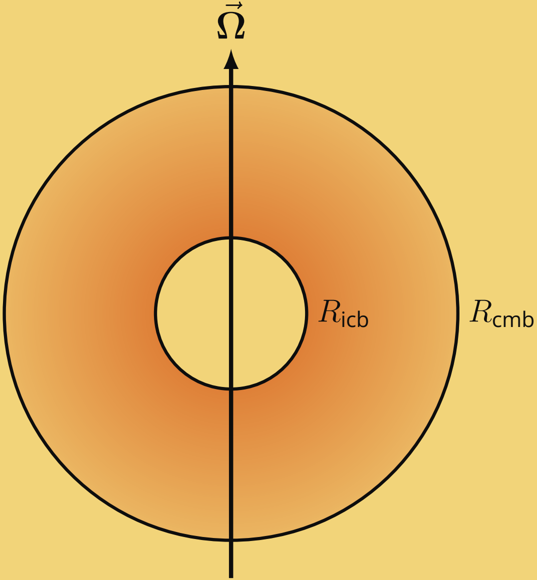
$$\omega^2 = N^2 \cos^2(\hat{\mathbf{v}}_g \cdot \hat{\mathbf{r}}) \tag{9}$$

and depends on the *Brunt-Väisälä frequency* N:

$$N^2 = \vec{\mathbf{g}} \cdot \boldsymbol{\nabla} \left(\frac{\rho'}{\rho_0} \right) \tag{10}$$

a measure for the stratification strength of the fluid.

Gravito-inertial waves.



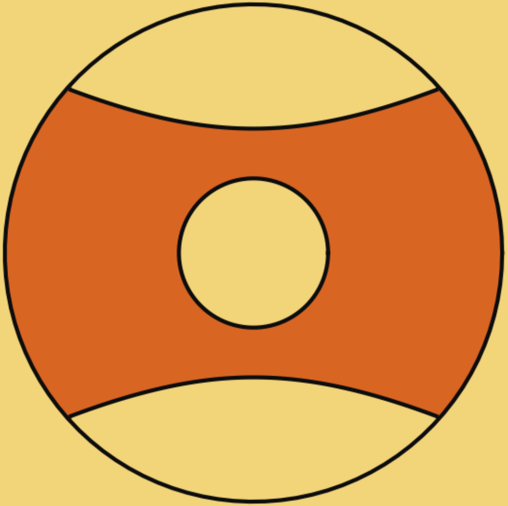
Combining inertial and gravity effects results in:

$$\partial_t \vec{\mathbf{u}} + 2\hat{\mathbf{z}} \times \vec{\mathbf{u}} = -\boldsymbol{\nabla} p - \frac{\rho'}{\rho_0} g \hat{\mathbf{r}} + \mathrm{Ek} \boldsymbol{\nabla}^2 \vec{\mathbf{u}} \,, \tag{11}$$

$$\partial_t \rho' = -\vec{\mathbf{u}} \cdot \boldsymbol{\nabla} \rho_0 \tag{12}$$

Solutions to these equations are called gravito-inertial waves.

Example of an inviscid gravito-inertial wave.



Buoyancy modifies the structure of the characteristics (Friedlander & Siegmann, 1981a):

- characteristics are curved
- characteristics are normal and reflect off a turning surface:

$$\frac{r^2 \cos^2 \phi}{\omega^2 - 1} + \frac{z^2}{\omega^2} = \frac{1}{N^2} \tag{13}$$

For $|\omega|/\Omega < N$ and $|\omega|/\Omega < 2$ the flow is confined between two hyperbolic surfaces.

Example of a viscous gravito-inertial wave

Reintroducing viscosity causes internal shear layers to appear again.

Summary of the waves in planetary fluid cores.

	dominant forces	**properties**
inertial waves	Coriolis	internal shear layers in the full spherical shell
Rossby wave	Coriolis	only exist in thin shells if $\ell \neq m$
gravity waves	buoyancy	flow becomes more horizontal when buoyancy force increases
gravito-inertial waves	Coriolis, buoyancy	internal shear layers in part of the spherical shell

Examples of stable layers in the solar system.

Numerically exploring rapidly rotating planets.

Numerical tool that solves the *linear* Navier-Stokes equation:

- for a viscous, incompressible and conductive fluid;
- enclosed within near-spherical boundaries;
- option: imposed magnetic field;
- option: imposed density gradient.

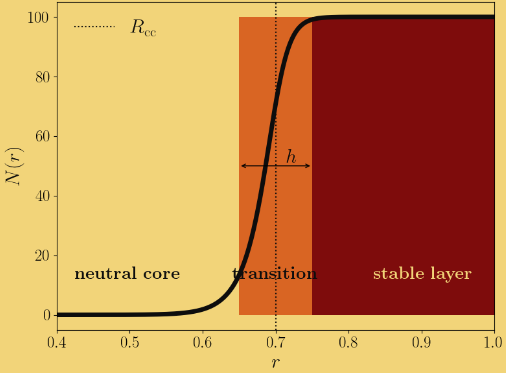


Fig 2. Example of a buoyancy profile for Kore, characterized by $R_{\text{ICB}} = 0.4$, $R_{\text{CC}} = 0.7$, $h = 0.2$ and $N(R_{\text{CMB}}) = N_{\text{CMB}} = 100$

Core flow induced by Mercury’s librations

Librations are periodic variations in the rotation rate of a planet or satellite.

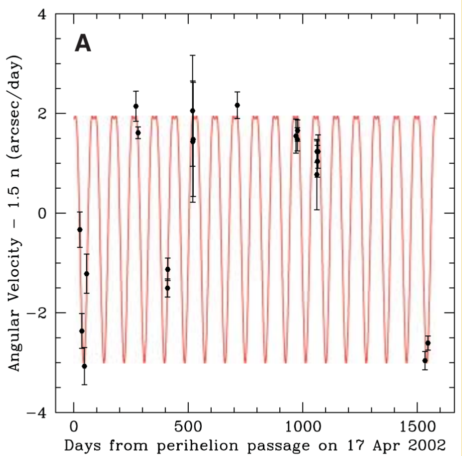


Fig. 3 Observed and numerically computed spin rate deviations. Figure taken from Margot et al. (2007).

Libration forcing as a boundary condition.

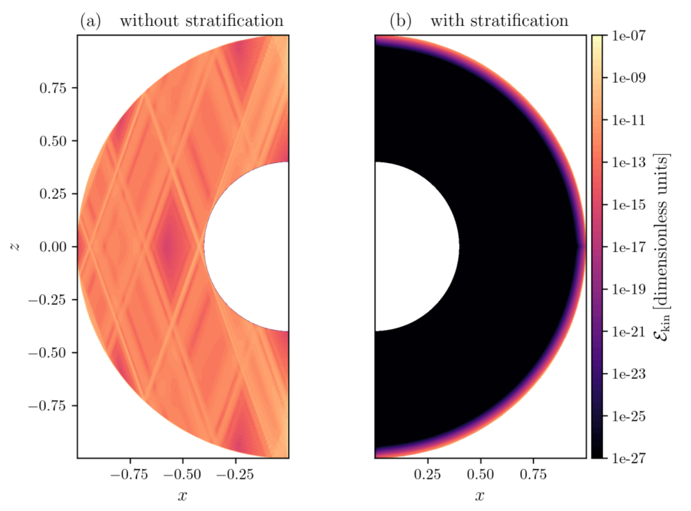
From Rekier et al. (2019) the libration can up to first order be represented as a superposition of three independent motions of the outer core boundary:

Azimuthal displacement of the triaxial boundary.

Tangential (*m*=*0*) forcing
Azimuthal displacement of the spherical boundary.

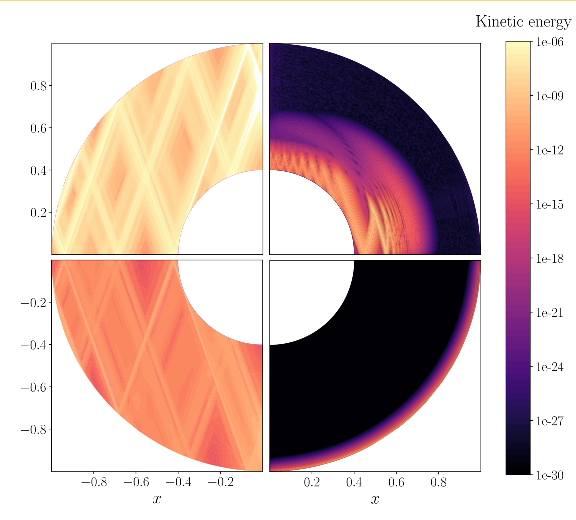
Radial (*m*=±*2*) forcing
Radial in- and outflow at the spherical boundary.

The stable layer suppresses radial flow motions.



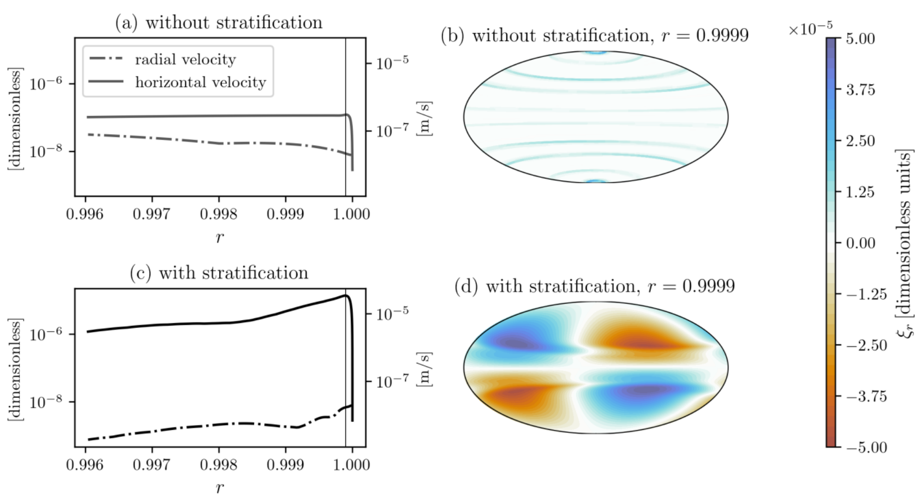
◄ Fig 4. Meridonial cut of the kinetic energy density ($\phi = 0$) of the core flow in response to the axial libration forcing in a core without stratification and a stratified core with $N_{\mathrm{CMB}} = 100$, $R_{\mathrm{ICB}} = 0.4$, $R_{\mathrm{CC}} = 0.7$ and $h = 0.2$ (Seuren et al., 2023, under review).

The stable layer suppresses resonance effects.



◀ Fig 5. Meridonial cuts of the kinetic energy density ($\phi = 0$) with librationally forced motions in the bottom row and eigensolutions in the top row, without stratification in the left column and with stratification in the right column.

The stable layer promotes a strong horizontal flow near the boundary.



◄ Fig 6. Radially averaged velocities and radial vorticity of the flow near the boundary in response to the radial libration forcing (Seuren et al., 2023, under review) .

Rossby waves in the Sun.

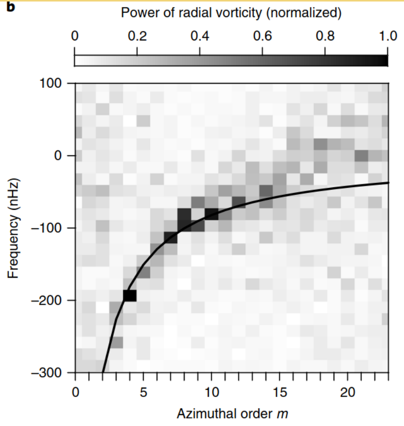


Fig 7. Power spectrum of the observed solar surface radial vorticity by Loptiën et al. (2018).

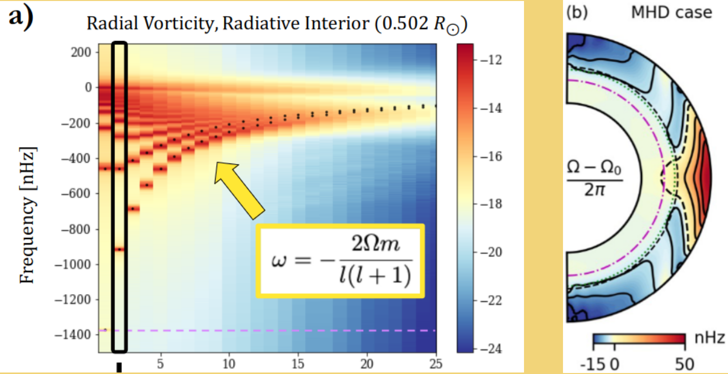
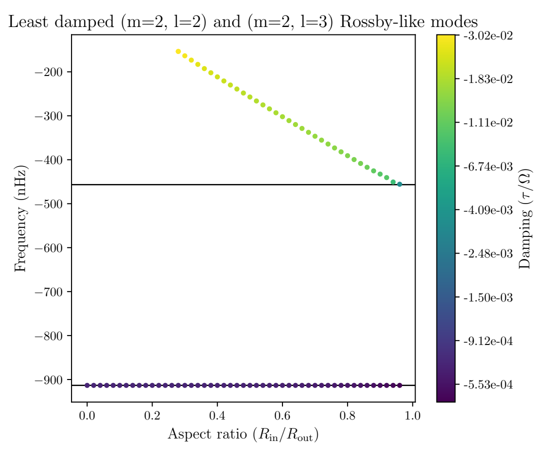


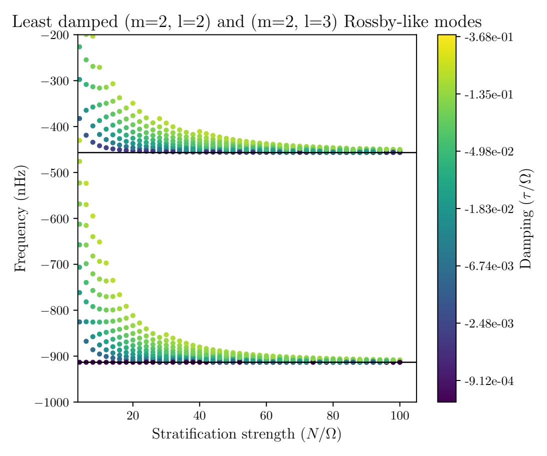
Fig 8. Power spectrum of the radial vorticity in the solar radiative interior (pink dash-dotted line on the right figure) from an MHD simulation by Blume et al. (2023), AGU poster.

No $m \neq \ell$ Rossby waves in the full sphere.



◄ Fig 9. Shell width versus frequency and damping of a sectoral ($m = \ell$) and a tesseral ($m \neq \ell$) Rossby-like wave.

Stable stratification serves as a rigid boundary.



◄ Fig 10. Stratification strength versus frequency and damping of a sectoral ($m = \ell$) and a tesseral ($m \neq \ell$) Rossby-like wave.

Potential weak and thin stably stratified layer near Earth’s outer core boundary.

Effect of a weak ($N/\Omega = 1$) stably stratified layer.

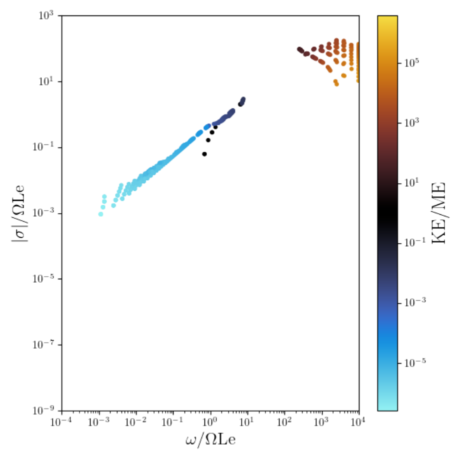


Fig 10. Eigenvalue spectrum **without** a stably stratified layer.

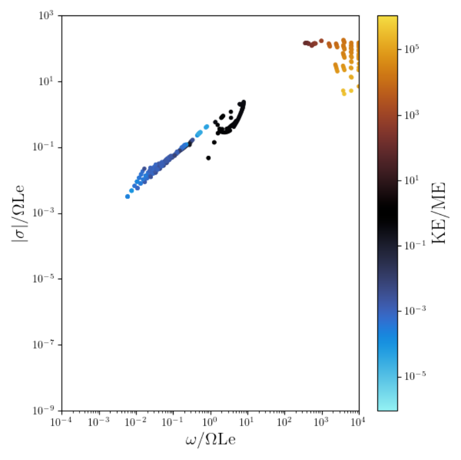
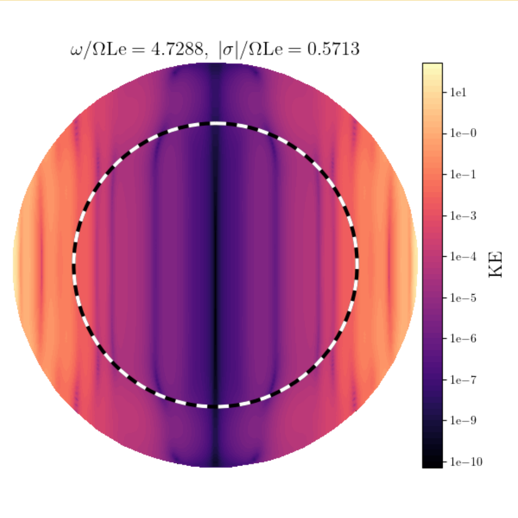


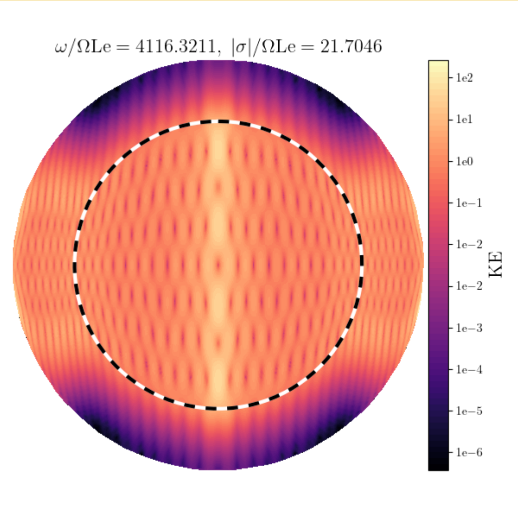
Fig 11. Eigenvalue spectrum **with** a stably stratified layer.

Columnar waves in the stratified layer.



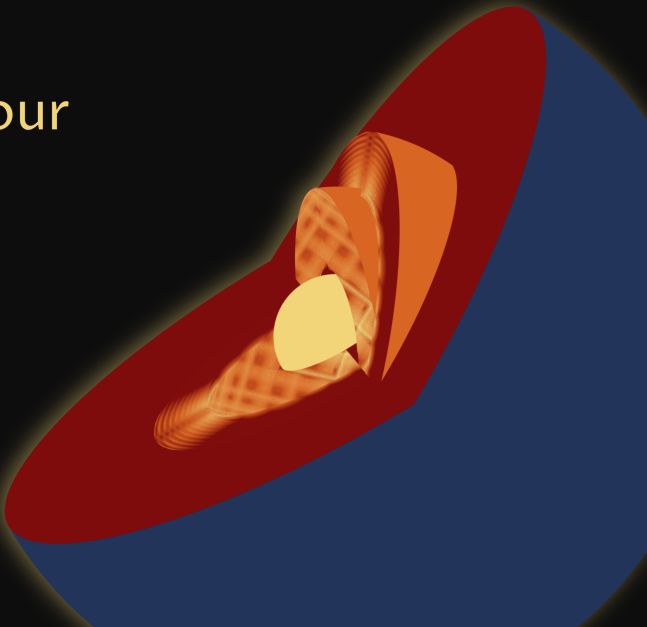
◄ Fig 11. Meridonial cut of the kinetic energy density of a wave propagating in the stably stratified outer layer.

Coupling between inertial and GI waves.



◄ Fig 12. Meridonial cut of the kinetic energy density of a inertial wave in the convective core coupled to a gravito-inertial wave in the stably stratified outer layer.

Thank you for your attention!



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