

The coupling between the polar motion and the spin precession of Titan

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Introduction

We develop, in an angular momentum approach, a consistent model for the polar motion and precession of Titan, a synchronously rotating satellite of Saturn. Titan harbors an internal global ocean and a thick atmosphere. We consider the solid layers to be rigid and we model the ocean's rotation as a Poincaré flow. We investigate the latitudinal free modes of rotation and the coupling between the forced solutions for polar motion and spin precession. We also compare the results of this new coupled model to those of existing decoupled rotation models that break the link between the two motions. The decoupled model for polar motion is taken from [1], whereas we have updated the model of decoupled spin precession of [2] to include the Poincaré flow.

1. Governing equations

The components of the rotation of a solid synchronous satellite can be described as solutions of a system of equations written in the satellite's Body Frame (BF):

$$\frac{d\vec{H}}{dt} + \vec{\Omega} \wedge \vec{H} = \vec{\Gamma}, \quad (1)$$

$$\frac{d\hat{p}}{dt} + \vec{\Omega} \wedge \hat{p} = 0, \quad (2)$$

with \vec{H} the angular momentum and $\vec{\Omega}$ the rotation vector, $\vec{\Gamma}$ the sum of the external torque by Saturn and of the atmospheric torque, and \hat{p} the unit vector along the Laplace pole expressed with respect to the Titan's BF. The first equation describes the change in angular momentum and is coupled to the second kinematic equation stating that the Laplace pole is fixed in inertial space [3].

We extend the model to the case of Titan divided into three layers: a solid icy shell, a liquid ocean, and a solid interior. The system of equation is then extended to three angular momentum equations (one for each

layer, taking into account the internal gravitational and pressure torques), and two kinematic equations (one for each solid layer, orienting them with respect to space). The equatorial components of each equation form a system of ten ordinary differential equations that can be written as

$$\dot{\vec{u}} + \mathbf{K}\vec{u} = \vec{T}, \quad (3)$$

where \vec{u} is the vector of the ten unknowns: the equatorial components of the variations in rotation of the shell, ocean, and interior with respect to the uniform rotation along the z -axis of the shell or interior BF, and of the unit vector along the Laplace pole expressed with respect to the shell and interior BFs. The vector \vec{T} contains the parts of the torques which do not depend on the variables to be solved for. The remaining parts of the torque are included in the product $(\mathbf{K}\vec{u})$, along with the cross product terms of the governing equations.

2. Free modes

The eigenvalues of \mathbf{K} correspond to the free frequencies of the latitudinal modes of rotation of the coupled model. We identify five different modes: the Chandler Wobble (CW) and the Interior Chandler Wobble (ICW) which are defined by analogy with Earth's studies, the Free Precession (FP) of the shell in space, the Free Ocean Nutation (FON) which replaces the Free Core Nutation of the Earth, and the Free Interior Nutation (FIN) which replaces the Free Inner Core Nutation of the Earth. The CW and ICW have long period behaviors in the rotating BFs and quasi diurnal behaviors in space. This is opposite for FP, FON, and FIN.

The model of decoupled spin precession, in which the polar motions of the solid layers are neglected, performs well in reproducing the FP, FON, and FIN periods of the coupled model (see Tab. 1). The decoupled model for polar motion, in which the solid layers are kept fixed in space, results in correct CW and ICW periods. The overall good correspondence between the

Table 1: Free periods (years) for a given interior structure of Titan, and for the different rotation models.

	Coupled model	Decoupled model for polar motion	Decoupled model for spin precession
T_{CW}	8.63	8.60	—
T_{ICW}	180.69	180.65	—
T_{FP}	9.14	—	9.10
T_{FON}	323.87	43.72	323.96
T_{FIN}	191.96	—	191.95

free modes of coupled and decoupled models indicates a weak coupling between polar motion and spin precession, even in the presence of a thick atmosphere.

3. Forced solutions

As a result of the weak coupling between the polar motion and the precession, the spin precessions of the three layers and the corresponding mean obliquities are mainly governed by the external gravitational torque of Saturn, and the polar motions of the solid layers are mainly governed by the angular momentum exchanges between the atmosphere and the surface. This quasi decoupling is illustrated for the case of an entirely solid Titan in Fig. 1.

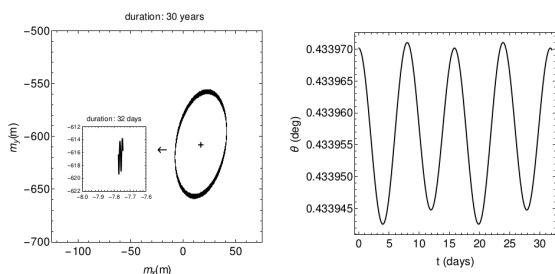


Figure 1: Left: Polar motion \vec{m} , as distance in meters at the surface of the satellite, which is mainly governed by the annual atmospheric torque. The offset due to the constant term of the atmospheric forcing is materialized by the cross markers. The zoomed region highlights the quasi diurnal component of the solution related to the external gravitational torque. Right panel displays the evolution over two diurnal cycles of the inertial obliquity θ . Its mean values and the semi diurnal variations are related to the external gravitational torque. The diurnal variations associated with the atmospheric torque are an order of magnitude smaller than the semi-diurnal variations.

The forced solutions of the coupled model correspond very well with the analytical solutions of decoupled models, which are easier to use in interpretations of observations from past and future space missions. Our results are to a good degree consistent with angular momentum or Hamiltonian coupled models available in the literature ([4, 5, 6]), validating our coupled model.

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