INFLUENCE OF A UNIFORM CORONAL MAGNETIC FIELD ON SOLAR P MODES

P. Vanlommel[1], A. Debosscher[2], J. Andries[2], and M. Goossens[2]

[1]Royal Observatory of Belgium, Ringlaan 3,B-1180 Brussels, Belgium

[2]Centre for Plasma Astrophysics, K.U.Leuven, Celestijnenlaan 200B, B-3001 Heverlee, Belgium

ABSTRACT

The influence of a constant coronal magnetic field on solar global oscillations is investigated for a simple planar equilibrium model. The model consists of an atmosphere with a constant horizontal magnetic field on top of an unmagnetized solar interior. The focus is on the possible resonant coupling of global solar oscillation modes to local slow continuum modes of the atmosphere and the consequent damping of the global oscillations. The physical process of resonant absorption of the acoustic modes with frequency in the cusp continuum is mathematically completely described by the ideal MHD differential equations which for this particular equilibrium model reduce to the hypergeometric differential equation.

Key words: Sun: oscillations, Sun: resonance.

1. INTRODUCTION

P modes are acoustic waves trapped in the upper layers of the Sun. The cavity is basically determined by the profile of the sound speed in the solar interior, but the oscillations can penetrate into the lower layers of the atmosphere and so are influenced by those layers. The atmospheric effects are not large, because the oscillations are predominantly determined by the internal structure of the Sun, but they are distinct. Magnetism is important in the Sun's atmosphere and is expected to influence the nature of global oscillation modes too.

In ideal MHD the resonant waves occur as a singularity in the differential equations governing the linear oscillations. The solutions diverge at the resonance point. The singularities in the ideal equations can be removed by including dissipative terms. Here, the model allows us to treat the resonance analytically without the need of including dissipative effects. The fact that the physical phenomenon of resonance is inherent and totally covered by the analytical solutions is the strength and beauty of this paper.

2. GENERAL LINEAR EQUATIONS

We adopt a system of Cartesian coordinates. The plane-wave approximation involves acoustic modes with sufficiently large values of the degree l and which are not affected by the curved shape of the Sun. Gravity is acting in the positive z-direction, the magnetic field is aligned horizontally in the x-direction. Linear isentropic harmonic perturbations with angular frequency ω and horizontal wave number $\mathbf{k} = k\mathbf{1}_x$ are imposed on an equilibrium model with a straight magnetic field in the x-direction. The two linear first-order differential equations for the z-component of the displacement, ξ_z and the Eulerian perturbation of total pressure, P are (Vanlommel and Goossens, 1999):

$$D\frac{\mathrm{d}\xi_z}{\mathrm{d}z} = C_1\xi_z - C_2 P \quad \text{and} \quad D\frac{\mathrm{d}P}{\mathrm{d}z} = C_3\xi_z - C_1 P_T, \tag{1}$$

with D, the coefficient of the highest order given by

$$D = \rho_0(v_s^2 + v_a^2)(\omega^2 - \omega_c^2) \tag{2}$$

Here, v_s is the sound speed, $v_a = B_0/\sqrt{\mu\rho_0}$ the Alfvén speed. $\omega_c = k^2 v_a^2 v_s^2/(v_s^2 + v_a^2)$ is the cusp frequency. The fact that ω_c^2 depends generally on z induces a mobile singularity in the differential equations at the zero of the coefficient D. Global oscillations with a frequency in this continuum can be resonantly absorbed. We investigate quasi modes which are nothing more than a weakly-damped natural coherent oscillation of an inhomogeneous system.

3. SOLUTIONS FOR THE LINEAR EQUATIONS

The choice of the equilibrium model with constant temperature and constant horizontal magnetic field for the atmosphere ($z < 0$) makes it possible for the global acoustic modes to resonate with a localized cusp wave. This happens if their frequency lies in the cusp continuum indicated as region II in Figure 1. The solutions of equation (1) for the z-component

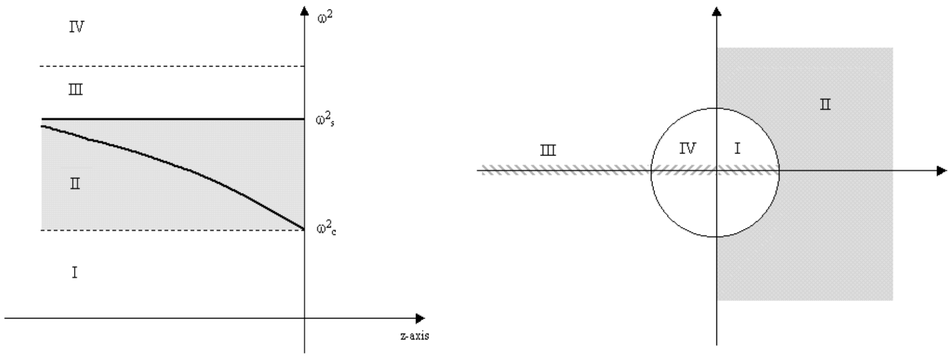

Figure 1. Domains of the complex ζ-plane connected with the frequency of the p mode

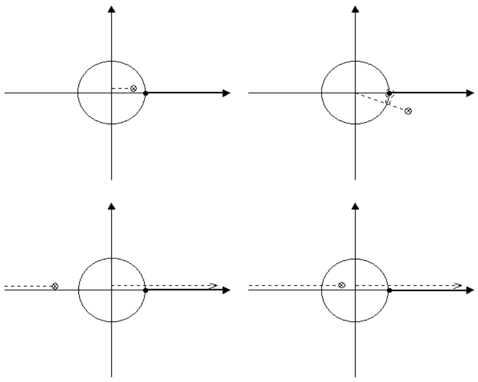

Figure 2. Integration paths

of the displacement, ξ_z and the Eulerian perturbation of total pressure, P are given in terms of the hypergeometric functions $F(p, q, r = p + q, \zeta)$. Here

$$\zeta = -A_1/A_2 e^{A_3 z}, \tag{3}$$

where A_1 and A_2 are functions of $\omega = (\omega_r, \omega_i)$, A_3 is a real number. So, ζ is complex and links the spatial coordinate z with a place in the complex plane according to the complex frequency ω. The hypergeometric solution around $\zeta = 0$ ($z = -\infty$) is given by a Gauss series (Abramowitz and Stegun). Analytical continuation in other domains of the complex plane and the fact that the hypergeometric function is multivalued in a circle around the singular point $\zeta = 1$ are the keys in the proper treatment of the solutions in the different domains of the atmosphere. In complex analysis, this multivalued character is represented by different Riemann sheet above each other, each sheet representing one value of the function F. While integrating from $z = -\infty$ to $z = 0$ in the atmosphere, we follow a corresponding continuous path in the complex plane. $z = -\infty$ corresponds to $\zeta = 0$, the end point of the integration path in the complex plane $\zeta(z = 0) = -A_1/A_2$ is determined by the frequency of the p mode. The correspondence is illustrated in Figure 1 where the right part show the complex ζ-plane. The circle is the unit circle. Figure 2 illustrates the four possible integration paths in the complex plane. The singular point $\zeta = 1$ is located at the intersection of the circle with the positive real axis. In the case of no resonance, we stay on the real axis. If resonances occur, we are forced to deviate from this real axis and go to the Riemann sheet below the sheet with the principal value (see right upper corner of Figure 2). This gives rise to the 'jump' of the physical parameters ξ_z and P while crossing the the resonance point. The strength here lies in the fact that the mathematics simply incorporate the jump conditions given by Sakurai et al. (1991). Both approaches are in essence the same. A thorough discussion about about dealing mathematically with the resonances can be found in Vanlommel et al. (2001).

In the convection zone ($z > 0$), the solutions for ξ_z and P are given by the confluent hypergeometric function U.

4. DISPERSION RELATION

The choice of the magnetic field allows for resonances in the corona. A variable cusp frequency creates a slow continuum; modes with a frequency within, can couple to localized slow waves. These modes are global damped oscillations of which part of the energy is carried away into the corona. This means that the frequency is complex, the imaginary part is responsible for the damping, while the real part is shifted. The effect of the resonant interaction decreases with the distance that the acoustic oscillation has to tunnel from the turning point at $z = 0$ in order to reach the resonant position.

$$\mathcal{P}_1(0)\mathcal{X}_2(0) - \mathcal{P}_2(0)\mathcal{X}_1(0) + \mathcal{X}_1(0)\mathcal{X}_2(0)g(\rho_c - \rho_p), \tag{4}$$

where $\mathcal{X}_{1,2}(0)$ and $\mathcal{P}_{1,2}(0)$ are the appropriate solutions for ξ_z and$P$ respectively, at the point $z = 0^-$ in the corona for the index 1 and at the point $z = 0^+$ at the top of the photosphere for the index 2. The last term of equation (4) is present because of the sharp boundary at $z = 0$. This dispersion relation is solved numerically.

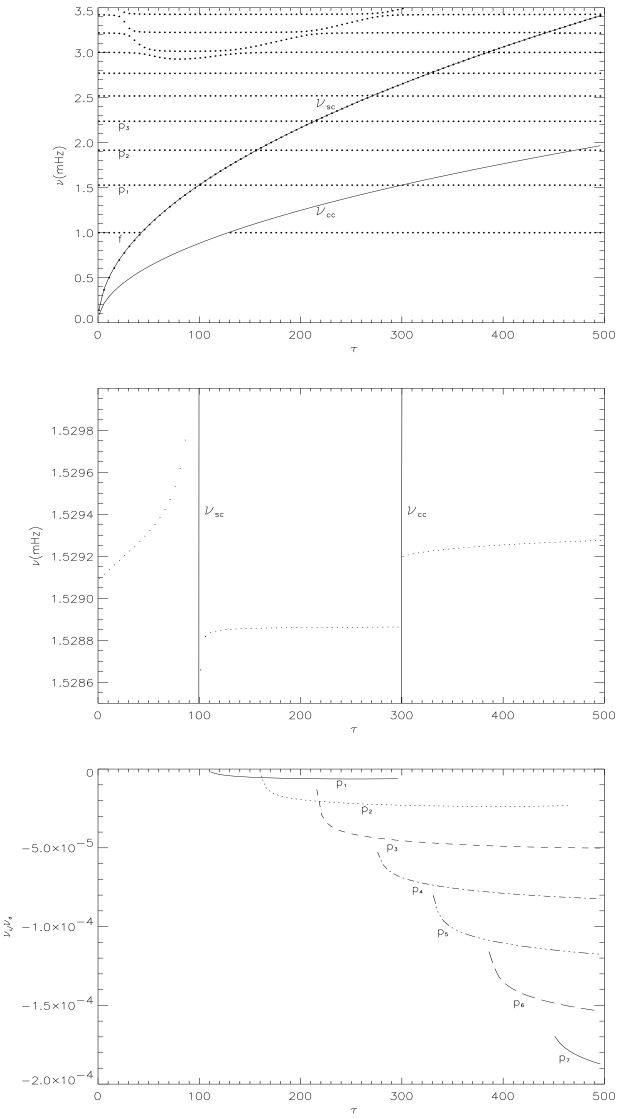

Figure 3. (a) Frequency as function of the temperature jump τ. (b) The frequency of the first p mode is shown as function of τ. The coronal value of the cusp and sound frequency at $z = 0$ is also given.(c) The ratio of the imaginary to the real part of the frequency of the p_i mode, $i = 1..5$, as function of the temperature jumpt τ. The magnetic field strength in the corona is 80 Gauss and the order $l = 100$.

5. RESULTS

Figure 3 shows the dependence of the modes on the temperature jump τ between the atmosphere and the solar interior (Vanlommel and Čadež, 1998). The first graph shows the real part of the frequency. The domain where resonances occur lies again between the two curves ν_{sc} and ν_{cc}. The curves representing the acoustic modes are bend when they reach ν_{sc} and follow the curve ν_{Sc} till the next mode of one order higher hits the curve of the sound frequency. In the higher part of the graph, several avoided crossings can be seen between the p modes. This resembles to the cut-off curve in the model where the Alfvén speed is constant in the corona (Vanlommel and Goossens, 1999). Evans and Roberts (1990) looked at the same model, but restricted their analysis implicitely to $\tau = 1$ and excluded so the possibility of resonances. Besides this, they didn't let the magnetic field increase above 100 Gauss and restricted all the calculations to area III of Figure 1.

Figure 3(b) shows how the p_1 mode is influenced by varying the temperature jump τ. When the coronal value of the sound frequency at $z = 0$ is reached, the mode avoids crossing with the mode coming from beneath. When the mode undergoes resonant coupling, the real part of the frequency is almost constant, changes are small compared to the changes of frequency when the mode is not absorbed. When the coronal cusp frequency at $z = 0$ is reached, we see again that the real part of the frequency is shifted. This can also be seen in Figure 3(b). The damping rate shown in Figure 3(c) increases in absolute value with increasing τ. Again, the larger the number i of the overtones p_i, the larger the absolute value of the damping rate. Figure 4 shows the Lagrangian displacement ξ_z in the corona, scaled to its value at $z = 0$. Since the hypergeometric functions are known in every point ζ (except at the singularity $\zeta = 1$), we can calculate the eigenfunction for all z, even in the resonance point z_c. The point $z = z_c$ is not a singular point because of the non-zero imaginary part of the frequency. We clearly see the 'jump' that the real and imaginary part of ξ_z make. To find the eigenfunctions in the region $[-\infty, z_c]$, the analytical continuation in the complex ζ plane of the hypergeometric functions in the unit circle around 0 is used. In the region $[z_c, 0]$, we look for solutions of the hypergeometric functions in the complex plane in the region outside the unit circle around 0, in that part where $Re(\zeta) > 0$ and in the Riemann sheet below the principal value.

6. CONCLUSIONS

We studied the resonant coupling of global solar oscillations to local slow continuum modes in the solar atmosphere. The equilibrium model is essentially that used by Evans and Roberts (1990). However, Evans and Roberts took $\tau = 1$ and excluded resonant coupling of the oscillation modes to the slow

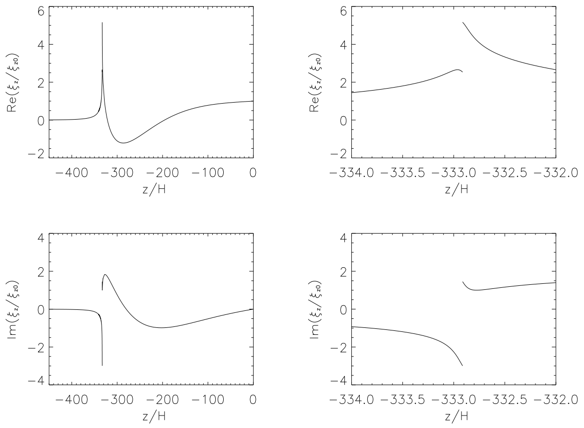

Figure 4. The z-component of the displacement of the third p mode as function of the spatial coordinate z/H in the corona. (a) and (b) show the real part, (c) and d) show the imaginary part, (b) and (c) are an enlargement of the eigenfunctions around the resonance point that is localized at $z_c/H = -332.91$. At the resonance point z_c, the functions are discontinuous and make a 'jump'. Other parameters: $B_c = 80$ Gauss, $l = 100$ and $\tau = 400$.

continuum modes of the atmosphere. This is in sharp contrast with our present study where the focus is on the resonant coupling. From a mathematical point of view, we allow for resonances investigating the whole complex ζ-plane in which the solutions of the standard hypergeometric differential equations can be found. The mathematics coincides in a remarkable way with the physical processes of resonance that global oscillations can undergo. The conclusions made by Evans and Roberts (1990) still hold when resonant coupling is absent ($\tau = 1$). By taking a value of τ different from 1, the conditions for resonance can be fulfilled. In the event of resonant coupling of the global modes to localized cusp waves, the conclusion differs. The real part of the frequency is not so sensitive any longer to parameters like the temperature jump τ. Besides this, the frequency is shifted downwards. The shift is of the order of $10^{-1}\mu$Hz. The imaginary part is responsible for the energy flow to the corona which can be heated in this way. When the frequency of the mode approaches the sound frequency in the corona at $z = 0$, the modes are very sensitive to the parameter τ. The frequency increases abruptly till the value of the frequency of the next mode is reached. The shift of the real part of the frequency is not negligible and within the limit of observational accuracy. This indicates that resonant interactions should definitely be taken into account when calculating the frequencies of the global solar oscillations.

A way to proceed now is to include Alfvén resonances. In this case, the differential equation cannot be brought to a known standard equation and therefore, has to be integrated numerically. The advantage to have analytical solutions is lost. Therefore, the cusp resonance is an important study case.

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