Доклади на Българската академия на науките
Comptes rendus de l'Académie bulgare des Sciences
Tome 54, No 9, 2001

PHYSIQUE
Physique de l'ionosphère

RECONSTRUCTION OF THE ELECTRON DENSITY PROFILE FROM THE TOTAL ELECTRON CONTENT USING UPPER TRANSITION LEVEL AND VERTICAL INCIDENCE SOUNDING MEASUREMENTS

S. M. Stankov, P. Y. Muhtarov

(*Submitted by Academician D. Mishev on April 25, 2001*)

Introduction. The total electron content (TEC) is one of the most important quantitative characteristics of the Earth's ionosphere and plasmasphere. The (vertical) TEC is defined as the integral of the electron density from the ground height up to the ceiling height (height of the transmitting satellite or infinity). The electron density above approximately 2000 km contributes little (less than 5%) to the integrated electron content and above the mean height of the plasmapause (25000 km) the contribution is negligible. All modern TEC measuring systems rely on the observation of signal phase differences or on pulse travel time and pulse shape measurements based on geostationary and orbiting satellites.

Given the electron density profile (i.e. the altitude distribution of the electron density), it is relatively easy to calculate the corresponding total electron content using quadrature formulae. The purpose of this paper is to present a novel approach to the solution of a long-standing problem in the space research – the reconstruction of the electron density distribution from the total electron content.

Mathematical formulation. There are possibilities of utilising the O^+–H^+ and O^+–He^+ transition levels as supplementary research tools [1, 2]. The main idea in the proposed reconstruction method is to use the upper (O^+–H^+) transition level as a reference point when solving the above stated inverse problem. The O^+–H^+ level is particularly suitable for the purpose because the relative abundance of hydrogen ions is a significant factor affecting the shape of the topside electron density profile. The transition level is used together with vertical incidence sounding (ionosonde) measurements. Required ionosonde measurements are the F2 layer ordinary critical frequency (foF2), the height of the peak electron density (HmF2) and the propagation factor M3000F2 (i.e. the ratio of the maximum usable frequency divided by the critical frequency). The peak electron density in the F region, NmF2, is computed from foF2 using the well-known relation $NmF2[m^{-3}]=1.24\times10^{10}\times(foF2[MHz])^2$.

The total electron content is split into a bottomside content, TEC_b, and a topside content, TEC_t, i.e.

$$TEC = TEC_b + TEC_t = \int_0^{HmF2} N_e(h)\ dh + \int_{HmF2}^{\infty} N_e(h)\ dh,$$

where $N_e(h)$ is the electron density at height h. The bottomside electron profile and corresponding bottomside electron content can be reliably calculated from foF2 and M3000F2 using established methods and models [3]. This study is focused on the determination of the topside electron profile.

The topside electron density profile is presented as a sum of its constituent oxygen and hydrogen ion density profiles. Further, the individual (oxygen and hydrogen) ion density distributions are approximated by the following analytical expression using Epstein functions [4]:

$$N_i(h) = N_i(h_mF2).sch^2\left(\frac{h - h_mF2}{2.H_i}\right) = \frac{4.N_i(h_mF2)}{\left(1 + \exp\left(\frac{h - h_mF2}{H_i}\right)\right)^2} \exp\left(\frac{h - h_mF2}{H_i}\right),$$

where $N_i(h)$ is the ion density at height h and H_i is the ion scale height. Therefore, the following "reconstruction" formula is proposed for calculation of the topside electron density profile:

$$(1)\quad \begin{array}{l} N_e(h) = N_{O+}(h_mF2)\,.sch^2\left(\frac{h - h_mF2}{2.H_{O+}}\right) + N_{H+}(h_mF2)\,.sch^2\left(\frac{h - h_mF2}{32.H_{O+}}\right), \\ h > h_mF2 \end{array}$$

where H_{O+} is the O^+ scale height. Several assumptions have been made [3]: first, the height of the O^+ density maximum is equal to the height of the H^+ density maximum; second, the neutral particles and ions have the same scale height; third, the hydrogen and oxygen ion temperatures are equal.

There are three unknown variables in the proposed formula – the oxygen and hydrogen ion densities at the peak height, i.e. $N_{O+}(h_mF2)$ and $N_{H+}(h_mF2)$, and the oxygen ion scale height H_{O+}. The following system is, therefore, constructed to determine the unknowns

$$N_{O+}(h_mF2) + N_{H+}(h_mF2) = N_mF2$$

$$N_{O+}(h_mF2)\ .\ sch^2\left(\frac{h_{tr} - h_mF2}{2\ .H_{O+}}\right) = N_{H+}(h_mF2)\ .\ sch^2\left(\frac{h_{tr} - h_mF2}{32\ .H_{O+}}\right).$$

$$TEC_t = 2.\ H_{O+}.\ N_{O+}(h_mF2) + 32\ .\ H_{O+}.\ N_{H+}(h_mF2)$$

The first equation represents the principle of plasma quasi-neutrality and the fact that at the peak height, the only ion densities that really count are the oxygen and hydrogen ion densities. The second equation denotes the fact that the hydrogen and oxygen ion densities are equal at the O^+–H^+ transition level; the upper transition level, h_{tr}, is determined empirically [5]. The third equation is obtained after integrating the proposed $N_e(h)$ "reconstruction" formula (1) from h_mF2 to infinity. The above system is solved numerically.

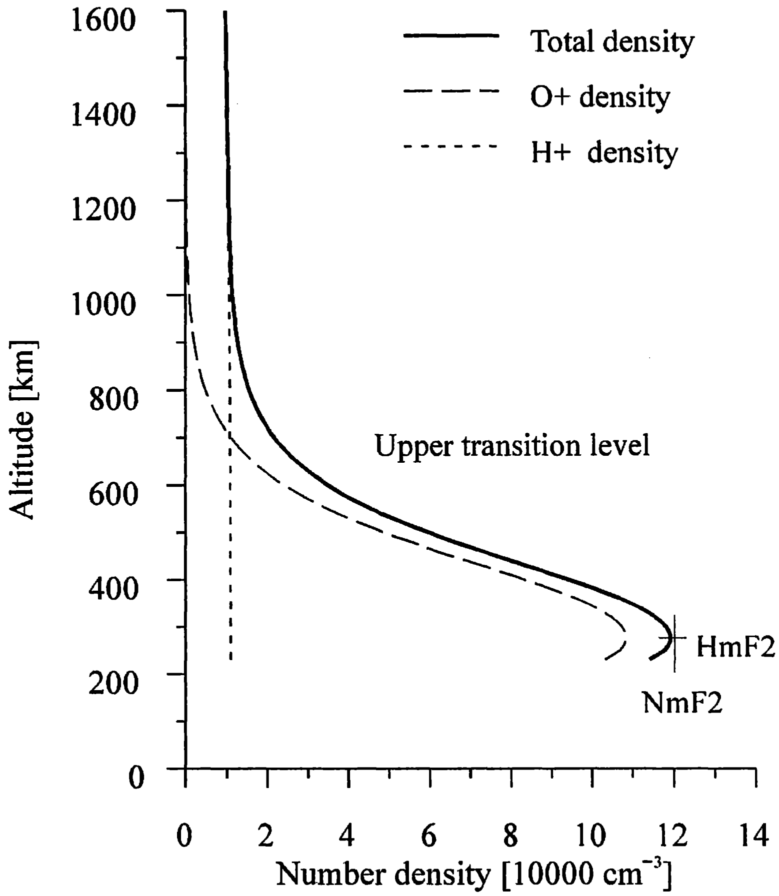

Fig. 1. Reconstruction of a topside electron density profile (solid line); the values of the required input parameters are $TEC = 7.1 \times 10^{16}$ [m^{-2}], foF2 = 3.10 [MHz], M3000F2 = 3.13, TL = 715 [km]

Results. The reconstruction is demonstrated using the following required data: total electron content (TEC $= 7.1\times10^{16}[m^{-2}]$), empirically obtained [5] upper transition level (h_{tr} = 715[km]), critical frequency (foF2 = 3.1[MHz]), and propagation factor M3000F2 = 3.13. The recovered topside electron density profile (solid line) is presented in Fig.1; the oxygen and hydrogen ion density profiles are also plotted.

The presence of helium ions is neglected as they probably have little effect on the electron density profiles and the integrated electron content under most conditions. However, the offered approach is flexible and allows the inclusion of the helium ions. The helium ion density could be introduced as a variable, and the system should be modified and expanded with an equation involving the O^+–He^+ transition level.

Conclusions. The described technique solves a difficult inverse problem – reproducing the electron density profile from its integral quantity, TEC. At present, routine measurements of electron density rely essentially on ground-based ionosonde soundings which can provide vertical profiles of the bottomside ionospheric electron density only. The offered reconstruction method delivers valuable information about the topside ionospheric and plasmaspheric density based on reliable routine satellite and ionosonde measurements. Possible applications include: (i) calibration of TEC measurements and verification of TEC models; (ii) evaluation of theoretical ionospheric and plasmaspheric models; (iii) plasma composition studies, etc.

REFERENCES

[1] BILITZA D. Adv. Space Research, **11**, 1991, No 10, 183–186. [2] STANKOV S. M. Bulg.Geoph.Journal, **25**, 1999, No 1–4, 95–116. [3] BILITZA D., K. RAWER, L. BOSSY, T. GULYAEVA. Adv. Space Research, **13**, 1993, No 3, 15–23. [4] RAWER K. Adv. Space Research, **2**, 1983, No 2, 183–186. [5] KUTIEV I., S. M. STANKOV, P. MARINOV. Adv. Space Research, **14**, 1994, No 12, 135-138.

Geophysical Institute
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 3
1113 Sofia, Bulgaria