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Understanding the effects of the core on the nutation of the Earth

Véronique Dehant*, Raphael Laguerre, Jérémy Rekier, Attilio Rivoldini, Santiago Andres Triana, Antony Trinh, Tim Van Hoolst, Ping Zhu

Royal Observatory of Belgium, 3 Avenue Circulaire, B-1180 Brussels, Belgium

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In this review paper, we examine the changes in the Earth orientation in space and focus on the nutation (shorter-term periodic variations), which is superimposed on precession (long-term trend on a timescale of years). We review the nutation modelling involving several coupling mechanisms at the core-mantle boundary using the Liouville angular momentum equations for a two-layered Earth with a liquid flattened core. The classical approach considers a Poincaré fluid for the core with an inertial pressure coupling mechanism at the core-mantle boundary. We examine possible additional coupling mechanisms to explain the observations. In particular, we examine how we can determine the flattening of the core as well as information on the magnetic field and the core flow from the nutation observations. The precision of the observations is shown to be high enough to increase our understanding on the coupling mechanisms at the core-mantle boundary.

1. Introduction

The relationship between the celestial frame and the terrestrial frames is complicated by the fact that the rotation and orientation of the Earth is subject to irregularities. The paper examines developments for improved model of Earth rotation and orientation at the sub-centimetre level.

The changes in the Earth orientation are caused by the gravitational attraction of the Sun and the Moon on the Earth, as well as many other factors that are progressively being identified by geodesists and geophysicists (in particular, the existence of a liquid core inside the Earth plays an important role). Because the Earth's shape can approximately be described as an ellipsoid flattened at its poles, the combined forces acting upon the Earth produce changes in both the speed of rotation and the orientation of the spin axis (Fig. 1). The term 'precession' describes the long-term trend of this latter motion (as seen on a timescale of several years), while 'nutation' is the name given to shorter-term periodic variations. These are the prime focus of the present paper. The precession of the Earth in space corresponds to about 50 arc seconds per year and the nutation amplitude is at the level of a few tens of arc seconds. The rotation axis of the Earth is moving in space at the speed of 1.5 km/year due to precession and has periodic variations with amplitudes at the level of 600 m (as seen from space in a plane tangent to the terrestrial pole). The present observations allow scientists to measure these at the sub-centimetre level.

Earth rotation changes, precession, and nutation are measured using VLBI (Very Long Baseline Interferometry), a technique that employs radio telescopes (Fig. 2) to observe extra-galactic radio sources such as quasars, which allows us to establish a celestial reference frame.

This geodetic technique is based on simultaneous recordings of radio emissions received from extra-galactic radio sources by a number of radio telescopes (5–20). The radio dish antennas, ranging from 3 to 100 m in diameter, form part of a worldwide VLBI network. They are situated at distances of hundreds to many thousands of kilometres from each other's. Each recording, electronically captured during the observation session, carries also, along with the radio emission received at the VLBI station, a time signal from the "station clock". These clocks are very high accuracy and stability atomic clocks, such as hydrogen maser clocks. Each

* Corresponding author.
E-mail address: v.dehant@oma.be (V. Dehant).

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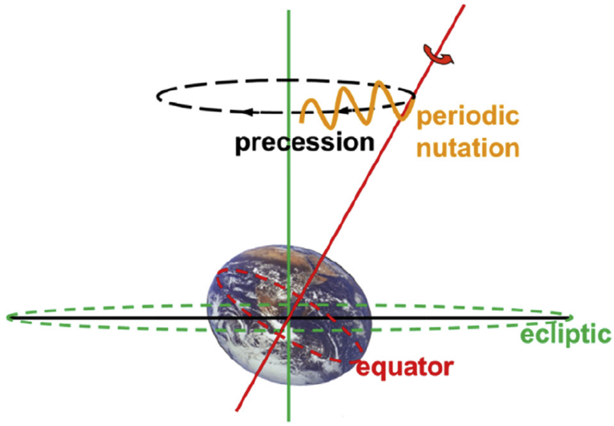

Fig. 1. Representation of precession and nutation of the Earth.

Fig. 2. VLBI antenna of Shanghai, China.

VLBI observing session lasts typically for a few hours to about 24 h. Emissions from many different radio sources (typically 100 sources) are recorded during one such session. The data are put on recording discs that are shipped, if not directly electronically transmitted, to a central processing site for analysis. One then evaluates the delay between the two arrival times of the same signal based on the expected identity of the temporal variations of the emissions recorded at two different stations. This determination is done by using a correlator/processor measuring with a precision at the level of 10 ps.

The VLBI observations are then compared with a model of the Earth orientation and rotation. The data have now been cumulated for over about 30 years. The data were showing large differences compared to theoretical nutation models at the beginning of the technique, leading to re-evaluation of the model. At present, they still show significant differences at the centimetre level (1 mas ≅ 3 cm). Fig. 3 show the differences between the VLBI observations and the results obtained from applying a theoretical model adopted by the IAU (International Astronomical Union) in 2000 (nutation) and 2006 (precession) and by the IUGG (International Union of Geodesy and Geophysics) in 2003 (see IERS Conventions [1–3]). If eventually corrected for the free FCN (Free Core Nutation) contribution, these differences are presently at one third of a mas level.

The adopted model depicts the Earth as a deformable object with a deformable inner core (viscoelastic central part of the Earth composed of a solid iron alloy), a liquid core (also composed of iron alloy), a deformable viscoelastic mantle (composed mainly of olivine and perovskite) (see Fig. 4), as well as oceans and an atmosphere. The adopted theoretical model is almost perfect as seen from the observed residuals. The gravitational torques exerted by other solar system bodies on the Earth are the cause of the nutations and can be precisely computed [4–6]. The response of the Earth to this forcing allows us to constrain interior properties and is modelled in terms of a transfer function [7–10]. The transfer function is the ratio between the amplitudes of the circular wobbles for the non-rigid and rigid Earth models and is independent of the amplitude of the torque. We may compute these ratios for the different frequencies of the forcing. The ratios are defined only for circular motions and are determined solely by the structure and properties of the non-rigid Earth. Transfer functions are of considerable utility as the nutations of the rigid Earth have been studied extensively in the literature [4–6] and highly accurate results are available. Once the transfer function is computed for the chosen non-rigid Earth model for any frequency of the excitation, the non-rigid Earth amplitude is simply obtained as the product of the transfer function with the rigid-Earth response at some chosen excitation frequency. The rigid Earth amplitudes (for an axially symmetric ellipsoidal model) depend on the dynamical ellipticity. A re-scaling according to the true non-hydrostatic dynamical ellipticity to be used can always be done as well. This computation ignores the external geophysical fluids and the small contributions from the oceans and atmosphere [11–15] have to be accounted for through further refinement of the model (for more detail [16,17]). The coupling mechanisms at the boundaries between the inner core, the liquid outer core, and the mantle are not yet understood or modelled accurately enough to be properly included inside a complete nutation model. The aim of our paper is to show, with a simple approach, how to get further insight into the Earth's interior, by coming up with a new model for nutation considering new advanced theories involving physical concepts for the non-rigid Earth that have not been taken into account previously.

The paper revisits the simple two-layer semi-analytical approach for better understanding and modelling the physical processes inside the Earth's core associated with Earth nutation and in particular at the CMB (core-mantle boundary) where uncertainties remain. The existence of a liquid core inside the Earth and all the coupling mechanisms at the core-mantle boundary play indeed an important role in nutation amplitudes. A mundane demonstration of the influence of the physical state of the interior on the rotation is that raw (liquid) and cooked (solid) eggs rotate differently. For the Earth, the identified mechanisms considered in the previous nutation model involve the flattening of the core and the fluid pressure and gravitational effect on that flattened boundary (see Ref. [18]) as well as a simple dipole and uniform magnetic field effect [10,19–21], because of their role in transferring angular momentum from the core to the mantle. By now we have understood that there are important roles from other components of the magnetic field [22–29], and possibly from the viscosity of the inner core [30–37], from the viscosity of the outer core [27–29], or from the core stratification [38–42] could have significant contributions to nutation, and that topography at the core-mantle boundary at the kilometre level plays an important role as well [43–45]. Seismic tomographic data have indeed pointed out the existence of valleys and mountains at the interface between the liquid core and the viscous deformable mantle down at about 2900 km below the surface. We also point out that the inertial waves associated with the topography at the core-mantle boundary may interact with the core flow produced by

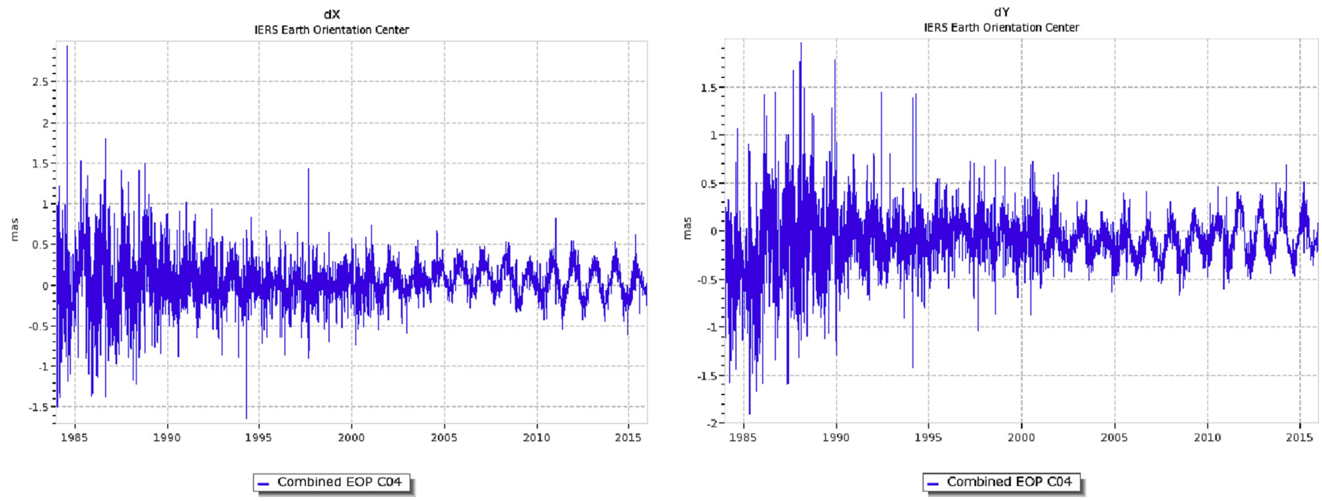

Fig. 3. Residuals in milliarc second (mas) with respect to the present-day used model of the position (*X*, *Y*) of the pole in space (from IERS Earth Orientation Center website http://hpiers.obspm.fr/eop-pc/).

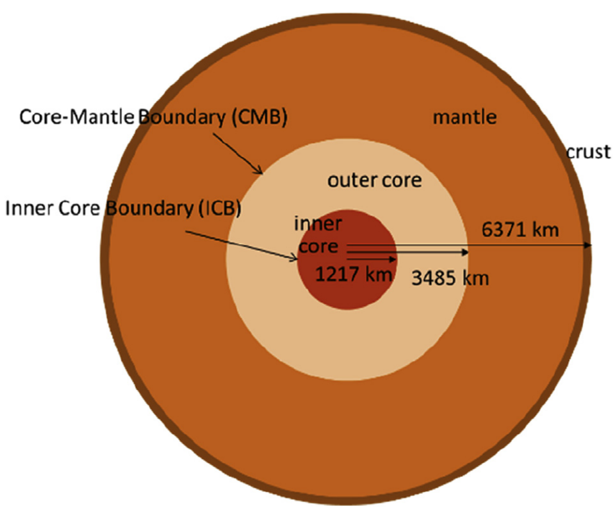

Fig. 4. Interior of the Earth as used for nutation computation.

nutations, and may thus induce resonance and out-of-phase ("dissipation") effects not taken into account yet in the present-day adopted models. In particular, we are presently working (in the frame of an ERC Grant called RotaNut for "Rotation and Nutation of a wobbly Earth") on the demonstration of the existence of shear layers that are building up in the fluid due to a perturbation of the Ekman boundary layer caused by topography at the CMB or any other phenomena.

While a detailed understanding of these coupling mechanisms could only come out from a fully-fleshed model, there is still some insight to be obtained by using deformation equation approach [17,46–63] or a simple semi-analytical approaches (see Refs. [64–68], or more recently [7–10,27,28]). The paper addresses this last approach in the frame of what we can learn from present-day observations. While the forcing is well known, the response of the Earth still needs further investigation in order to understand the next decimal place of the nutations. In particular, one of the latest developments were identified to be related to the existence of rotation dynamics in the liquid core inside the Earth. In that frame, working with a simplified Earth helps to better explain the role of the core in the future models.

2. Simple Liouville equations for nutation and wobble

In the simplest approach, the Earth nutation can be computed from the convolution (corresponding to a product in the frequency domain) between a rigid-Earth nutation forcing at different frequencies and a transfer function for its response to this forcing at the same frequencies, as explained in the introduction. Due to the kinematic relations connecting nutation and wobble, the respective transfer functions and the nutation computations are identical. Note first that the existence of a circular nutation is associated with every circular (retrograde diurnal) wobble and vice versa; the frequency of the nutation (σ_n, using the subscript *n* here for nutation) is algebraically greater than that of the associated wobble frequency (ω_w, using the subscript *w* here for wobble)by 1 cpsd (cycle per sidereal day) Ω_0, the uniform Earth rotation.

$$\sigma_n = \omega_w + \Omega_0 \tag{1}$$

This important general relation is simply a consequence of the Earth's diurnal rotation in space.

For a circular nutation $X + iY$ or η of frequency σ_n, we have:

$$X + iY = \eta = \eta(\sigma_n)e^{i(\sigma_n t + \chi_n)} \tag{2}$$

where $\eta(\omega_n)$ is the nutation amplitude and χ_n the nutation phase. The wobble *m* can be written in the same form:

$$m = m(\omega_w)e^{i(\omega_w t + \chi_w)} \tag{3}$$

where $m(\omega_w)$ is the wobble amplitude and χ_w the wobble phase.

Introducing the Euler angles, one can relate a body-fixed reference frame to an inertial reference frame. The time derivatives of the Euler angles represent combinations of rotations of the three body-fixed axes with respect to the inertial frame and are related to the rotation of the planet and its precession and nutation. The relation between the components (in the body-fixed reference frame) of the rotation vector and the time derivatives (in inertial space) of the Euler angles is given by the Euler kinematic equations (see Refs. [17,67] for instance). Because the figure axis is defined in terms of the moment of inertia or equivalently of the internal planetary mass elements and because these mass elements are rotating about the rotation axis, a variation in this rotation axis with respect to an inertial reference frame also leads to an identical variation in the figure axis with respect to the inertial reference frame. In addition, any periodic variation (nutation) in the orientation of the rotation or figure axis in space is accompanied by aperiodic variation of the rotation axis with respect to the figure axis (polar motion or wobble, m), and vice versa. During a wobble, both the figure and the rotation axes have to change their orientation in inertial space (otherwise, the planet's spin angular momentum cannot be conserved). Mathematically, from the first two Euler kinematic equations (see Refs. [17,67] for instance), we obtain:

$$\eta(\sigma_n) = \frac{\Omega_0}{\sigma_n} m(\omega_w) \tag{4}$$

Consider now two Earth models, one rigid and the other non-rigid, both of which are forced by the same torque. Let the wobbles of the rigid (R) and non-rigid (NR) Earth models be $m_R(\omega_w)$ and $m_{\mathrm{NR}}(\omega_w)$, respectively. The transfer function may be taken from their ratio:

$$T_w = \frac{m_{\mathrm{NR}}(\omega_w)}{m_{\mathrm{R}}(\omega_w)} \tag{5}$$

We have similarly for the nutation transfer function:

$$T_n = \frac{\eta_{\mathrm{NR}}(\sigma_n)}{\eta_{\mathrm{R}}(\sigma_n)} \tag{6}$$

The frequency domain version of the kinematical relation [17,67] are valid for any type of Earth model, rigid or non-rigid. We have then:

$$T_n = \frac{\eta_{\mathrm{NR}}(\sigma_n)}{\eta_{\mathrm{R}}(\sigma_n)} = T_w = \frac{m_{\mathrm{NR}}(\omega_w)}{m_{\mathrm{R}}(\omega_w)} \tag{7}$$

Or equivalently,

$$\eta_{\mathrm{NR}}(\sigma_n) = T_n \eta_{\mathrm{R}}(\sigma_n) = T_w \eta_{\mathrm{R}}(\sigma_n) \tag{8}$$

which allows us to compute the non-rigid Earth nutation from the rigid-Earth nutation and the wobble transfer function.

The wobbles are described by the Liouville equations expressing the angular momentum conservation in a uniformly rotating frame. For the simplified case of an elastic Earth with a liquid core and considering that the wobble frequency is near −1 cpsd, one obtains, in diurnal approximation, the momentum conservation of the global Earth:

$$-\Omega_0 m + \frac{A_f}{A}\omega_w m_f = \frac{i\Gamma}{\Omega_0^2 A} \tag{9}$$

where A, A_m and A_f are the global Earth, mantle, and core moments of inertia respectively (with $A = A_f + A_m$), where m_f is the core wobble, and Γ the gravitational torque acting on the Earth.

Similarly, the angular momentum conservation of the core is

$$-\Omega_0 m + \left(\beta - \alpha_f\right)\Omega_0 m_f = 0 \tag{10}$$

which represents the equation for the Poincaré fluid. α_f is the core dynamical flattening and β is the so-called compliance accounting for the deformation of the Earth at the core-mantle boundary.

The retrograde diurnal free wobble, ω_{FCN},corresponding to the Free Core Nutation (FCN, σ_{FCN}) is then provided solving the above equation for ω_w considering $\Gamma = 0$:

$$\omega_{FCN} = -\Omega_0\left[1 + \frac{A_m}{A}\left(\beta - \alpha_f\right)\right] = -\Omega_0 + \sigma_{FCN} \tag{11}$$

$$\sigma_{\mathrm{FCN}} = -\Omega_0 \frac{A_m}{A}\left(\beta - \alpha_f\right) \tag{12}$$

The solutions of the Liouville equations for an elastic Earth with a liquid core in diurnal approximation provide expressions for the wobble of the global Earth and the core:

$$m = \frac{i\Gamma}{\Omega_0^2 A_m}\left(1 + \frac{\frac{A_f}{A}(\Omega_0 + \omega_{\mathrm{FCN}})}{\omega_w - \omega_{\mathrm{FCN}}}\right) = \frac{i\Gamma}{\Omega_0^2 A_m}\left(1 + \frac{\frac{A_f}{A}\sigma_{\mathrm{FCN}}}{\sigma - \sigma_{\mathrm{FCN}}}\right) \tag{13}$$

$$m_f = \frac{\Omega m}{\frac{A_f}{A}\sigma_{\mathrm{FCN}}} \tag{14}$$

where we see that there is a resonance effect in the wobble induced by the FCN and that the core wobble is much larger than the mantle wobble.

3. Liouville equations for an elastic Earth with a liquid core highly coupled with the mantle in diurnal approximation

The previous solutions for the wobbles are solutions for a flattened Earth with inertial pressure on the flattened core-mantle boundary and gravitational effects between the core and the mantle. In reality, there are other coupling mechanisms and dissipation at the core-mantle boundary related to the existence of viscosity, electromagnetic filed, and topography in addition to the hydrostatic flattening.

This adds a coupling torque $\Gamma_{\mathrm{CMB}}^{\mathrm{torque}}$ at the core-mantle boundary. The Liouville equation for the global Earth is the same as previously but the equation for the core is then different:

$$-\Omega_0 m + \frac{A_f}{A}\omega_w m_f = \frac{i\Gamma}{\Omega_0^2 A} \tag{15}$$

$$-m + \left(\beta - \alpha_f\right)m_f = \frac{i\Gamma_{\mathrm{CMB}}^{\mathrm{torque}}}{\Omega_0^2 A_f} = \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)m_f \tag{16}$$

where $\Gamma_{\mathrm{CMB}}^{\mathrm{torque}} = -i\Omega^2 A_f(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}})m_f$, expresses the fact that the torque depends on the differential rotation between the core and the mantle, m_f. K_{CMB}^{Re} is the real part of the coupling constant, and K_{CMB}^{Im} is the imaginary part.

The normal mode is similar in this case than in the non-dissipative case presented in the previous section, except for the existence of the coupling constant:

$$\omega_{\mathrm{FCN}} = -\Omega_0\left[1 + \frac{A}{A_m}\left(\beta - \alpha_f - \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)\right)\right] \tag{17}$$

$$\sigma_{\mathrm{FCN}} = -\Omega_0 \frac{A}{A_m}\left(\beta - \alpha_f - \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)\right) \tag{18}$$

The solutions of the Liouville equations take exactly the same expressions as before:

$$m = \frac{i\Gamma}{\Omega_0^2 A_m}\left(1 + \frac{\frac{A_f}{A}(\Omega_0 + \omega_{\mathrm{FCN}})}{\omega_w - \omega_{\mathrm{FCN}}}\right) = \frac{i\Gamma}{\Omega_0^2 A_m}\left(1 + \frac{\frac{A_f}{A}\sigma_{\mathrm{FCN}}}{\sigma - \sigma_{\mathrm{FCN}}}\right) \tag{19}$$

$$m_f = \frac{\Omega m}{\frac{A_f}{A}\sigma_{\mathrm{FCN}}} \tag{20}$$

but with σ_{FCN} given above, involving the coupling constant $K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}$. It must be noted that when the hydrostatic equilibrium of the Earth is abandoned, the dynamical flattening α_f has a non-hydrostatic contribution:

$$\alpha_f = \alpha_f^{\mathrm{hydr}} + \alpha_f^{\mathrm{nonhydr}} \tag{21}$$

The total torque $\Gamma_{\mathrm{CMB}}^{\mathrm{torque}}$ involves a combination of a topographic torque (the result of the applied pressure from the core on a non-spherical boundary), a viscous torque (coming from the drag exerted by the viscous fluid at the CMB), and an electromagnetic torque (arising from the stretch of the magnetic field lines by the core motion) so that the coupling constant is the sum of these effects:

$$K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}} = \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)_{\mathrm{electromag}} + \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)_{\mathrm{viscous}} + \left(K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}\right)_{\mathrm{topo}} \tag{22}$$

For the electromagnetic part, using the weak field approximation, it is possible to linearly relate the $(K_{\mathrm{CMB}}^{\mathrm{Re}})_{\mathrm{electromag}}$ and $(K_{\mathrm{CMB}}^{\mathrm{Im}})_{\mathrm{electromag}}$, as done in Ref. [7]. In that case, the two constants are very close to one another. As shown in Ref. [33], the viscous coupling must be very small. If the topographic coupling is neglected, the amplitude of the magnetic field that is needed to explain the observed imaginary part of the coupling constant must be very high. However, this situation has recently been challenged [43,45]. The third coupling mechanism, proportional to $(K_{CMB}^{Re} + iK_{CMB}^{Im})_{topo}$, is mainly related to core flow acting on the CMB topography. It is reasonable to think that we have a kind of symmetry in this torque as well, except if there would be an anomalistic antisymmetric CMB topography (a particularly large bump). In general, it is reasonable to think that the topographic torque plays a role in the coupling between the core and the mantle and that its real and imaginary parts remain of the same order of magnitude as for electromagnetic coupling mechanism. The topographic torque depends on the topography itself, which is difficult to evaluate precisely. If the topographic torque would be small, it remains possible but highly speculative that the magnetic field would be very large. Wu and Wahr (1997) [45] have evaluated the topographic torque to be of the same order as the electromagnetic torque.

Other "dissipation" mechanisms can be considered in addition. These can be found in examining the dynamics of the liquid core more carefully. Inertial waves and associated shear layers induced by the fluid in the core or at the core-mantle boundary is one of the mechanisms that we suspect to be the most efficient. Although the viscous dissipation associated with these layers decreases with the Ekman number, the Ohmic dissipation increases in these localized shear flow regions. The presence of inertial waves can then influence dramatically the flow dynamics, particularly at the extremely low Ekman numbers associated with the Earth and other planetary cores. This is the main objective of our present research.

4. Comparison with observations

By fitting the nutation model to recent VLBI observations, Zhu et al. [69] (this issue, see as well Refs. [27,33,70–72]) have obtained several values for the Earth parameters, as it was done previously [15,27–29,32]. In particular, they have determined the value of the combination $\alpha_f + K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}} = 0.002676 + i0.000019$. This combination appears directly in the FCN frequency determined from the forced nutation observations. Neglecting as usual the viscous torque, if one considers, as classically considered in the case of electromagnetic coupling, that $K_{\mathrm{CMB}}^{\mathrm{Re}} \approx iK_{\mathrm{CMB}}^{\mathrm{Im}}$ (the real and imaginary parts of the topographic torque can be considered of the same order of magnitude as well), this leads to $\alpha_f + K_{\mathrm{CMB}}^{\mathrm{Re}} = 0.002657$. Considering that the hydrostatic value of the dynamical flattening α_f^{hydr} is 0.002548 (as computed from a rotating Earth with internal properties determined from seismology), this leaves us with $\alpha_f^{\mathrm{nonhydr}} = 0.000109$ for the non-hydrostatic contribution, which corresponds to an increase of the equatorial radius of about 370 m.

5. Conclusion and perspectives

We have presented a simple introduction to nutation modelling, using the Liouville angular momentum conservation equations for a two-layer Earth with a liquid core. This classical approach considers, as a first approximation, a Poincaré fluid for the core with an inertial pressure coupling mechanism at the core-mantle boundary. Additional coupling mechanisms are superimposed and modelled in terms of coupling constants. These coupling constants are the basic Earth parameters that enter in the Earth transfer function for nutations and that are thus determined from the recent VLBI observation series. Nutation amplitudes are mostly dictated by the frequency and damping of the FCN (and FICN extend). The other two rotational eigen frequencies, the Chandler wobble and the Inner Core Wobble frequencies, have small impact on nutation and are essentially independent of the coupling strengths at the core boundaries and so are the corresponding resonance coefficients. Suffice it to say that one may expect the amplitudes of nutations with frequencies close to the FCN (or the FICN) eigen frequency to be affected most by the introduction of the coupling strengths $K_{\mathrm{CMB}}^{\mathrm{Re}} + iK_{\mathrm{CMB}}^{\mathrm{Im}}$ (and $K_{\mathrm{ICB}}^{\mathrm{Re}} + iK_{\mathrm{ICB}}^{\mathrm{Im}}$). The finding of Mathews et al. [10] and confirmed by Zhu et al. [69] (this issue)is that the contributions from the CMB and ICB couplings to the real (in-phase) and imaginary (out-of-phase) parts of the retrograde annual and retrograde 18.6 year nutation amplitudes are all quite significant, of the order of 0.5 mas. We found that, if the imaginary part and the real part of the sum of the electromagnetic torque and the topographic torque are of the same order of magnitude, the increase of the equatorial radius of the core is at the level of 370 m. This is the most significant result of this paper.

Furthermore, recent estimates of the coupling constants suggest that topographic coupling may, in addition to electromagnetic coupling, explain the observed "dissipation" reflected in the coupling constant at the core-mantle boundary.

Acknowledgements

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Véronique Dehant works at the Royal Observatory of Belgium, where she is Responsible for the Operational Directorate "Reference Systems and Planetary Science". She is also Extraordinary Professor at the Université Catholique de Louvain. She was or is involved as Principal Investigator (PI) or Co-Principal Investigator (Co-PI) or Co-Investigator (Co-I) of several radioscience space experiments aiming at planetary geodesy and at physics of the interior of terrestrial planets.She is Academician (Royal Academy of Belgium; Science class) since 2010, Doctor Honoris Causa of Paris Observatory since 2014, "Membre Etranger" of the Académie des Sciences de Paris since 2015, and was awarded with several prizes, including the Descartes Prize of the European Union in 2003. She has taken several international responsibilities at President level such as President of the "Geodesy" Section of the American Geophysical Union. In 2015, she has obtained a European Research Council (ERC) Advanced Grant, with the project RotaNut: Rotation and Nutation of a wobbly Earth.She is author of more than 445 publications including 155 refereed publications, and of more than 975 oral or poster presentations at the present date (end 2016).